• C6 – Introduction to Energy
Energy in interactions between two particles

- Total energy involved in an interaction between two particles is defined as

\[ E = K_1 + K_2 + V(r) \]

- \( K_1 \) is the kinetic energy of one particle
- \( K_2 \) is the kinetic energy of the other particle
- \( V(r) \) is the potential energy that depends on the distance between the particles
- \( E, K_1, K_2, \) and \( V(r) \) are all **scalar** quantities
Total Energy

(a) \[ E = \text{all } V \]

(b) \[ E = \text{some } K + \text{smaller } V \]

(c) \[ E = \text{more } K + \text{still smaller } V \]
The energy of a multiparticle system is the sum of the kinetic energies of the particles and potential energy of the interaction between the particles.

\[ E = K_1 + K_2 + K_3 + \ldots + V(r_{12}) + V(r_{13}) + V(r_{23}) + \ldots \]

- The kinetic energies of particles 1, 2, and 3
- The potential energy of the interaction between particles 1 and 2
Energy conserved for ISOLATED system

- Internal interactions between particles involves only transfers between kinetic and potential energies.
- The system’s total energy is not changed by these INTERNAL interactions.
- External interactions can change the kinetic or potential energies of the particles in our system.
- Total energy is conserved only for ISOLATED systems.
Kinetic Energy

- Energy of a moving particle
  \[ K \equiv \frac{1}{2} m v^2 \]
  \[ K \equiv \frac{(mv)^2}{2m} = \frac{p^2}{2m} \]

- SI unit of energy
  \[ J = \text{kg} \cdot \text{m}^2 / \text{s}^2 \]
Ignore earth’s kinetic energy

- Ratio of kinetic energy of objects
  \[
  \frac{K_2}{K_1} = \frac{\frac{p_2^2}{2m_2}}{\frac{p_1^2}{2m_1}} = \frac{m_1}{m_2}
  \]

- If \(m_1\) is something of normal mass, and \(m_2\) is the earth, this implies that \(K_2 \ll K_1\)

- When something of normal mass interacts with the earth, the earth’s kinetic energy can be completely ignored.
We can only measure potential energy difference

- We can measure the change in potential energy by comparing their positions and kinetic energies at two different times.

\[ V(r_f) - V(r_i) = K_{1i} - K_{1f} + K_{2i} - K_{2f} \]

\[ \Delta V = \Delta K_1 + \Delta K_2 \]

- \( i \) is the initial time and \( f \) is the next time a measurement is taken.
Potential Energy Functions - C7

- Electromagnetic Interaction
  \[ V(r) = +k \frac{q_1 q_2}{r} \]
  reference separation: \( V(r) = 0 \) when \( r = \infty \)

- Gravitational Interaction
  \[ V(r) = -G \frac{m_1 m_2}{r} \]
  reference separation: \( V(r) = 0 \) when \( r = \infty \)

- Potential Energy of a Spring
  \[ V(r) = \frac{1}{2} k_s (r - r_0)^2 \]
  reference separation: \( V(r) = 0 \) when \( r = r_0 \)
Gravitational potential energy

- One equation for the gravitational interaction potential energy

\[ V(z) = mgh \]

- \( z \) is the distance measured vertically from a reference point
- This equation is only valid close to the earth’s surface
Negative Energy?

- Only true of Potential energy
  - Can be positive or negative with respect to a reference separation

\[ V(z) = mgz \]

- Freedom to define reference separation
- Kinetic energy is always positive
Conservation of Energy – Hidden Forms

• In future chapters, our equation for energy will include additional terms that represent energy transfers due to interactions.

• One example is friction, it seems to remove energy from the system. It’s actually channeling the energy into hidden forms of energy that we can’t see, such as the vibration of individual atoms.

\[ 0 = \Delta K_1 + \Delta K_2 + \Delta V_g + \Delta U \]
Solving problems using framework

- Translation
  - 2 pictures – initial state and final state
  - Reference frame axes on each picture
  - Labels for masses, velocities and separations

- Conceptual model diagram
  - Interaction diagram – like momentum
  - Include symbols inside system to show changes in kinetic and potential energies

- Master equation

\[ 0 = \Delta K_1 + \Delta K_2 + \ldots + \Delta V + \ldots \]
Problem (A Penny from Heaven) Imagine that you accidentally drop a penny from the observation deck of the Empire State Building (height ≈ 1200 ft = 370 m). Ignoring air friction, what will be the penny’s approximate speed when it hits the ground? Why would you guess that throwing things off the observation deck is forbidden?
Translation

Before:

- Penny with mass \( m \) (\( v_i = 0 \))

After:

- \( z_f = 0 \)

Known:
- \( v_i = 0 \)
- \( z_i = 370 \text{m} \)
- \( z_f = 0 \)
- \( g = 9.8 \text{N/kg} \)

Conceptual Model Diagram

- Penny \( \Delta K_p \)
- Gravity \( \Delta V \)
- Friction (small)
- Earth \( \Delta K_e \)
- Air

CoE: floats in space

0 = \( \Delta K_p + \Delta K_e + \Delta V \)

\[ = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + 0 + \frac{1}{2} m g z_f^2 - \frac{1}{2} m g z_i^2 \]

Earth is very massive
Example C7.3

A Friend Who Suddenly Pops Up

Problem A friend of yours sits on a coiled spring that has been compressed to a length 55 cm shorter than its normal length and then held at that length. What is this spring’s spring constant $k_s$ if, when the spring is suddenly released, your friend flies into the air 2.0 m above his or her initial position?

Translation

Initial:
Before:

Final:
After:

Known:

$v_i = \text{mag}(\vec{v}_i) = 0$
$v_f = \text{mag}(\vec{v}_f) = 0$
$z_i = 0, z_f = 2.0 \text{ m}$
$r_0 - r_f = h = 0.55 \text{ m}$

We can look up $g$
Translation

Initial:
Before:

Final:
After:

Known:
$v_i = \text{mag}(\vec{v}_i) = 0$
$v_f = \text{mag}(\vec{v}_f) = 0$
$z_i = 0, z_f = 2.0 \text{ m}$
$r_0 - r_i = h = 0.55 \text{ m}$

We can look up $g$.

Conceptual Model Diagram

Floats in space

- Person $\Delta K_p$
- Gravity $\Delta V_g$
- Spring $\Delta V_s$
- Air friction (small)
- Earth $\Delta K_e$

CoE: system floats in space

$0 = \Delta K_p + \Delta K_e + \Delta V_g + \Delta V_s$

$= \frac{1}{2} mg f_0^2 - \frac{1}{2} mg f_0^2 + 0 + mgz_f - mgz_f + 0 - \frac{1}{2} k (r_i - r_0)^2$

$m = 55 \text{ kg}$

- Estimate
- Earth is very massive
- Friend is close to the earth's surface

$r_i - r_0 = h$
Group Problems

- C6S.5
- C7B.2
- C7S.3
- Each group can go at their own pace to complete the problems.