• C2 – Vectors
• C3 – Interactions transfer momentum

Solutions to HW
• When your homework is graded and returned, solutions will be available.
• Download ProbViewer 1.4
  • [www.physics.pomona.edu/sixideas/sicpr.html](http://www.physics.pomona.edu/sixideas/sicpr.html)
• Password for C1 daily HW problems
  • phys130cn
• Corrections are due on Thursday, 9/17
Homework Solutions

- **Given:**
- **Find:**
  - Draw a picture
  - Label units
  - Show your work!!!!!
- *Don’t need to rewrite the question.*

Quizzes

- Quiz at the start of each class
- Questions will come from:
  - Two-minute Problems
  - Chapter Exercises
Two Minute Problems – C2T.4

• A reference frame drawn on a sheet of paper has its $y$ direction oriented toward the top of the sheet and its $x$ direction toward its right edge. What direction must the $z$ direction point if the frame is right handed?

• A. Outward, perpendicular to the plane of the paper.
• B. Inward, perpendicular to the plane of the paper.
• C. Diagonally to the lower left, in the plane of the paper.
• D. Diagonally to the lower right, in the plane of the paper.
• E. Vertically downward in the plane of the paper.
• F. Other (specify).

Two Minute Problems – C2T.1

• The magnitude and direction of a vector can be described without using a reference frame, true (T) or false (F)?
Two Minute Problems – C2T.9

Consider the two vectors shown in figure C2.12. The sum of these vectors points most nearly

- A. Up.
- B. Down.
- C. Right.
- D. Left.

Two Minute Problems – C2T.11

Consider the two vectors shown in figure C2.12. To change \( \vec{u} \) into \( \vec{w} \), one would have to

- A. Multiply by -1.
- B. Multiply by 120.
- C. Add the vector \( \vec{u} + \vec{w} \).
- D. Add the vector \( \vec{w} - \vec{u} \).
- E. Add the vector \( \vec{u} - \vec{w} \).
- F. Do none of the above.
Two Minute Problems – C3T.2

- Imagine that a 1.0-kg cart traveling rightward at 1.0 m/s hits a 3.0-kg cart at rest. Afterward, the smaller cart is observed to move leftward with a speed of 0.75 m/s. What impulse did the collision give the smaller cart at the expense of the larger?
  - A. None; the larger cart was at rest and so had no momentum to give.
  - B. None; the lighter cart gave an impulse to the more massive cart, not the other way around.
  - C. 0.75 kg m/s leftward.
  - D. 1.00 kg m/s leftward.
  - E. 1.75 kg m/s leftward.

Two Minute Problems – C3T.3

- Imagine that a moving cart (cart A) hits an identical cart (cart B) at rest. Cart B remains at rest, and cart A rebounds with a speed equal to its original speed. Cart B must have participated in some other interaction during the collision process, true (T) or false (F)?
Two Minute Problems – C3T.9

- Which of the following statements involving vectors are correct? Answer T if it is correct and F if it is not (be prepared to identify the error if you answer F).
  - a. \( \vec{v}_{tot} = \vec{v}_1 + \vec{v}_2 \) implies that \( p_{tot} = p_1 + p_2 \).
  - b. \( \vec{p} = m\vec{v} \) implies that \( m = \frac{\vec{p}}{\vec{v}} \).
  - c. If \( \vec{v} = [0, -5.0 \text{ m/s}, 0] \), then \( \vec{v}_y = -5.0 \text{ m/s} \).
  - d. If \( \vec{v} = [0, -5.0 \text{ m/s}, 0] \), then \( \vec{v}_y = -5.0 \text{ m/s} \).
  - e. If \( \vec{v} = 5.0 \text{ m/s} \) and \( m = 2.0 \text{ kg} \), then \( \vec{p} = 10\text{ kg m/s} \)

Equality of Two Vectors

- Two vectors are **equal** if they have the same magnitude and the same direction.
- All of the vectors shown in the diagram at right are equal.
Adding Vectors Graphically

- Draw the vectors “tip-to-tail”
- The resultant is drawn from the origin of \( \vec{A} \) to the end of the last vector
- Measure the length of \( \vec{R} \) and its angle

Adding Multiple Vectors

- Tip-to-tail for all vectors
- The resultant, \( \vec{R} \), is still drawn from the origin of the first vector to the end of the last vector
Subtracting Vectors

- Special case of vector addition
- If $\vec{A} - \vec{B}$, then use $\vec{A} + (-\vec{B})$
- Continue with standard vector addition procedure

Unit Vectors

- A unit vector is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction (x, y, and z) and have no other physical significance
- The symbols for x, y, z are $\hat{x}, \hat{y}, \text{and} \hat{z}$
- They form a set of mutually perpendicular vectors
Adding Vectors Using Unit Vectors

- Using \( \vec{R} = \vec{A} + \vec{B} \)
- Then
  \[
  \vec{R} = (A_x \hat{x} + A_y \hat{y}) + (B_x \hat{x} + B_y \hat{y})
  \]

  \[
  \vec{R} = (A_x + B_x) \hat{x} + (A_y + B_y) \hat{y}
  \]

  \[
  \vec{R} = R_x \hat{x} + R_y \hat{y}
  \]

  \[
  \vec{R} = \begin{bmatrix} A_x \\ A_y \end{bmatrix} + \begin{bmatrix} B_x \\ B_y \end{bmatrix} = \begin{bmatrix} A_x + B_x \\ A_y + B_y \end{bmatrix}
  \]

- and so \( R_x = A_x + B_x \)
- and \( R_y = A_y + B_y \)

\[
R = \sqrt{R_x^2 + R_y^2} \quad \theta = \tan^{-1} \frac{R_y}{R_x}
\]
Adding Vectors Using Unit Vectors – Three Directions

- Using $\vec{R} = \vec{A} + \vec{B}$
  $\vec{R} = (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) + (B_x \hat{x} + B_y \hat{y} + B_z \hat{z})$
  $\vec{R} = (A_x + B_x) \hat{x} + (A_y + B_y) \hat{y} + (A_z + B_z) \hat{z}$
  $\vec{R} = R_x \hat{x} + R_y \hat{y} + R_z \hat{z}$

- $\vec{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$, $\vec{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$, $\vec{R} = \begin{bmatrix} A_x + B_x \\ A_y + B_y \\ A_z + B_z \end{bmatrix}$

- $R_x = A_x + B_x$, $R_y = A_y + B_y$ and $R_z = A_z + B_z$

- $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$
  $\theta_x = \tan^{-1} \frac{R_x}{R}$

Think-Pair-Share

- (a) In unit-vector notation, what is the sum of $\vec{a} + \vec{b}$, where
  $\vec{a} = (4.0 \text{ m}) \hat{x} + (3.0 \text{ m}) \hat{y}$
  $\vec{b} = (-13.0 \text{ m}) \hat{x} + (7.0 \text{ m}) \hat{y}$

- What are the (b) magnitude and (c) direction of $\vec{a} + \vec{b}$ (relative to $\hat{x}$)?
Think-Pair-Share

- Repeat the problem using column vector notation:
  what is the sum of $\vec{a} + \vec{b}$, where

  \[ \vec{a} = (4.0 \text{ m}) \hat{i} + (3.0 \text{ m}) \hat{j} \]
  \[ \vec{b} = (-13.0 \text{ m}) \hat{i} + (7.0 \text{ m}) \hat{j} \]

Displacement

\[ \Delta \vec{r} = \vec{r}_2 - \vec{r}_1 \]
\[ \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} - \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix} \]
**Velocity**

- Change in position in a small time interval

\[
\vec{v} = \frac{d\vec{r}}{dt} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \frac{1}{dt} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} dx/dt \\ dy/dt \\ dz/dt \end{bmatrix}
\]

- Column vector


**Momentum**

- "Quantity of motion"
- Vector
- Defined as

\[
\vec{p} \equiv m\vec{v}
\]

- in terms of mass and velocity
- Units are kg\cdot m/s

General Physics 23
**Impulse**

*Amount of momentum that an interaction (A) transfers to either particle during a short interval*

\[ d\vec{p}_A \]

**Change in momentum**

*Multiple interactions contribute their own impulses resulting in a change in momentum*

\[ d\vec{p} = [d\vec{p}]_A + [d\vec{p}]_B + [d\vec{p}]_C + \ldots \]
Impulse - skill building problem
• What is the impulse on an 8-kg ball rolling at 2 m/s when it bumps into a pillow and stops?

Financial Analogy
• Particle’s momentum is a person’s net financial worth
• Interaction is a financial interaction
  • Increases one’s net worth at the expense of the other’s
• Impulse is a check that is written or received
• Momentum in general is like money
Force

- Rate at which momentum flows

\[ \overrightarrow{F}_A = \frac{\left[ d\vec{p} \right]_A}{dt} \]