N1 – Newton’s Laws and N2 - Vector Calculus
Newton’s First Law

- Total momentum of an isolated system is conserved.

\[ Mv_{CM} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_N = \vec{p}_{tot} \]

- In the absence of external interactions, an object’s or system’s center of mass moves at a constant velocity.
Newton’s Third Law

- A tiny impulse $[\frac{d\vec{p}}{dt}]$ out of A is the same as $-\frac{d\vec{p}}{dt}$ flowing into A

$$\vec{F}_B = \left[\frac{d\vec{p}}{dt}\right] = -\left(\frac{d\vec{p}}{dt}\right) = -\vec{F}_A$$

- When objects A and B interact, the force of the interaction exerts on A is equal in magnitude and opposite in direction to the force that it exerts on B.
Newton’s Second Law

- The net external force on an object causes its mass to accelerate

\[ \vec{F}_{net, ext} = \frac{d\vec{p}_{tot}}{dt} = M \frac{d\vec{v}_{CM}}{dt} \equiv M\vec{a}_{CM} \]

- One of the most important equations in physics!
Classification of Forces

All macroscopic physical forces

Forces arising from long-range interactions
- Gravitational forces $\vec{F}_g$
- Long-range electromagnetic forces

Forces arising from contact interactions
- Spring forces $\vec{F}_{sp}$
- Contact between solid and fluid (liquid or gas)
  - Drag forces $\vec{F}_D$
    (oppose motion of object relative to the fluid)
  - Lift forces $\vec{F}_L$
    (act perpendicular to object's motion relative to fluid)
  - Thrust forces $\vec{F}_{Th}$
    (exerted when propeller, jet engine, etc. forces fluid to move)
  - Pressure forces $\vec{F}_P$
    (exerted when fluid is compressed while confined in some way)

Forces acting between two solid objects
- Normal forces $\vec{F}_N$
  (the part of contact force acting perpendicular to interface between solids)
- Frictional forces $\vec{F}_{KF}$
  (the part of contact force acting parallel to interface between solids)
- Static friction $\vec{F}_{SF}$
  (prevents surfaces from moving relative to each other at the interface)
- Kinetic friction $\vec{F}_{KF}$
  (opposes motion of surfaces relative to each other)

- Tension forces $\vec{F}_T$
  (resist separation of objects)

Figure N1.1
Free-Body Diagrams

- Draw interaction diagram/conceptual diagram
- Draw object alone
- For each force, draw a dot where the force is applied and an arrow representing magnitude and direction
- Arrows only for forces
- Every force arrow reflects an interaction
Free-body diagram - book on a table

- Interaction/conceptual diagram
- Draw object
- Draw dot for each force
- From dot draw direction of each force
Group Problems
Free body diagrams

Object slides at constant speed without friction.

Object slows due to kinetic friction.

Object slides without friction.

The object is pulled by a force parallel to the surface.

The object is pushed by a force applied downward at an angle.

The ball is rising in a parabolic trajectory.
Vector Calculus

- Why?
  - In order to better understand velocity and acceleration
  - Newton had to invent calculus to explain his theory!

- Review in appendix NA and NB at end of Volume N
Time Derivative of a Vector

- The time derivative is defined as

\[
\frac{df}{dt} \equiv \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}
\]

- The difference between \( \frac{df}{dt} \) and \( \frac{\Delta f}{\Delta t} \) is that
  - The first expression is at an instant in time
  - The second expression is for a short time interval
Derivative of a Vector

- The components of the time derivative vector are the ordinary time derivatives of the components of that vector.

\[
\frac{dq}{dt} \equiv \begin{bmatrix} \frac{dq_x}{dt} \\ \frac{dq_y}{dt} \\ \frac{dq_z}{dt} \end{bmatrix}
\]
Definition of Velocity

- The velocity vector is the derivative of the displacement vector.

\[ \vec{v} \equiv \frac{d\vec{r}}{dt} \equiv \frac{\text{small displacement}}{\text{short time interval}} \]

\[ \vec{v} \equiv \frac{d\vec{r}}{dt} \equiv \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} \]

\[ v = \text{mag}(\vec{v}) = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \]
Velocity - Instantaneous and Average

Figure N2.1

Direction of motion at A

Object’s trajectory

\[ \Delta \vec{r} \]

\[ \vec{r}(t) \]

\[ \vec{r}(t + \Delta t) \]

(a)

Direction of motion at A

Object’s trajectory

\[ \Delta \vec{r} \]

\[ \vec{r}(t) \]

\[ \vec{r}(t + \Delta t) \]

(b)

Direction of motion at A

Object’s trajectory

\[ \Delta \vec{r} \]

\[ \vec{r}(t) \]

\[ \vec{r}(t + \Delta t) \]

(c)

Direction of \( \vec{v} \) at middle of interval

Direction of \( \vec{v} \) at beginning of interval

\[ \Delta \vec{v} \]

(Remember that \( \Delta \vec{r}/\Delta t \) has the same direction as \( \Delta \vec{r} \).)
Definition of Acceleration

- An object’s acceleration at an instant is a vector that expresses *how rapidly* and *in what direction* its velocity vector is changing at that instant.

\[ \vec{a} \equiv \frac{d\vec{v}}{dt} \equiv \begin{bmatrix} \frac{dv_x}{dt} \\ \frac{dv_y}{dt} \\ \frac{dv_z}{dt} \end{bmatrix} \]
Motion Diagrams

Visualize this:

(a)

(b)

1  2  3  4  5

(c)

1  2  3  4  5

\[ \vec{v}_{12} \Delta t \quad \vec{v}_{23} \Delta t \quad \vec{v}_{34} \Delta t \quad \vec{v}_{45} \Delta t \]

\[ \Delta \vec{v} \Delta t \approx \vec{a}_2 \Delta t^2 \]
Motion Map - Acceleration

\[ \vec{a}_2 \Delta t^2 \quad \vec{a}_3 \Delta t^2 \quad \vec{a}_4 \Delta t^2 \]
Motion Diagram Example
Group Activity - Make a Motion Map

- Throw the ball vertically upward
  - Not too high
- Using LoggerPro, capture the throw
- On paper, draw a motion map
- How does the velocity change?
  - Draw velocity vectors
- Draw acceleration vectors
Group Problems

- N1S.2
- N2B.6
- N2B.8