Finish N4 - Motion from Forces

- Kinematic Curves
- Group Problems
  - N4S.9
  - N4S.10
Forces from Motion - Overview

- Chapters 5 – 9
  - How to use Newton’s second law to solve practical and real world problems
  - This chapter starts with the simplest case of motion – *no* motion
N5 - Statics

- An extended body at rest is a statics problem

\[ \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... = 0 \]

\[ m\ddot{a} = 0 \]
Example Problem Ex N5.1

- An object of mass $m$ hangs in the earth’s gravitational field from a string attached to a hook embedded in the ceiling. What is the magnitude of the downward force that the hanging mass exerts on the hook?
Example Problem – N5B.1

If the diagonal string makes an angle $\theta$ from the vertical, what is the magnitude of the tension force it exerts on the hanging mass, as a fraction or multiple $mg$?
The Rotating Person

(a) Weight
(b) \( \vec{\omega}_f > \vec{\omega}_i \)

Stool free to rotate around a vertical axis
Cross Product

\[ \text{mag}(\vec{u} \times \vec{w}) = uw \sin \theta \]

(a) \( \text{mag}(\vec{w}_\perp) = w \sin \theta \)

(b) \( \text{mag}(\vec{u}_\perp) = u \sin \theta \)

\[ \vec{q} = \vec{u} \times \vec{w} \]

\[
\begin{bmatrix}
q_x \\
q_y \\
q_z
\end{bmatrix} =
\begin{bmatrix}
u_y w_z - u_z w_y \\
u_z w_x - u_x w_z \\
u_x w_y - u_y w_x
\end{bmatrix}
\]

Figure C13.3
The right-hand rule that defines the direction of \( \vec{u} \times \vec{w} \).
The lengths of the hour and minute hands of a clock are 4 cm and 6 cm, respectively. If the vectors $\vec{u}$ and $\vec{w}$ represent the hour and the minute hands, respectively, then $\vec{u} \times \vec{w}$ at 5 o’clock is

- A. 24 cm² up
- B. 24 cm² down
- C. 21 cm² up
- D. 21 cm² down
- E. 12 cm² up
- F. 12 cm² down
Angular Momentum

- Angular momentum is the cross product between the particle’s displacement from O and its momentum

\[ \vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} \]
Angular Momentum - con’t

- For a particle moving along a circular path about O, the angular momentum is the moment of inertia times it’s angular velocity.

\[ L \equiv \text{mag}(\vec{L}) = mrv_{\perp} = rm(r\omega) = mr^2\omega = I\omega \]

\[ \vec{L} = I\vec{\omega} \]

- The angular momentum of an symmetric extended object that is spinning around its center of mass is also given by this equation.
Translation vs Rotation

**Translation**

\[
\vec{p} = m \vec{v}
\]

\[
\vec{F} = m \vec{a}
\]

\[
\dot{\vec{F}} = \frac{[d\vec{p}]}{dt}
\]

**Rotation**

\[
\vec{L} = \vec{r} \times \vec{p} = I \vec{\omega}
\]

\[
\tau = \vec{r} \times \vec{F} = I \vec{\alpha}
\]

\[
\dot{\tau} = \frac{[d\vec{L}]}{dt}
\]
Torque

- The torque may also be calculated from the displacement of a force from the axis of rotation

\[
\vec{\tau} = \left[ \frac{d\vec{L}}{dt} \right] = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}
\]

*Figure C13.9*
Pushing on a door with one’s hand.
Statics - Force and Torque

- Sum of the forces = 0
- Sum of the torques = 0

\[ \vec{F}_1 + \vec{F}_2 + \ldots = 0 \]
\[ \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \ldots = 0 \]
Statics and Torque

Figure N5.3
Where should we attach the wire to suspend this rod?
Statics and Torque

Plank, mass \( m \)

\[ r_1 = \frac{L}{5} \quad r_2 = \frac{2L}{5} \]

**Figure N5.4**

How to support a plank.
Example Problem – N5B.3

Imagine that we place a 10-g weight at the 10-cm mark on the uniform meter stick, and we find that the meter stick now balances at the 45 cm mark. What is the mass of the meter stick?
Solving Force-from-Motion Problems

- Translation
  - Drawing
  - Define symbols – masses, distances, angles, etc.
  - List of known values to right of drawing
  - Define coordinate system

- Conceptual Model
  - Add acceleration arrow to translation diagram
  - Free-body diagram
  - Newton’s second law
  - Newton’s second law in column vector form
  - Circle unknowns
  - Balloons for assumptions/approximations
Solving Statics Problems

- Write $a = 0$ and $\tau = 0$ on translation diagram
- On free-body diagram, indicate origin, and locations where forces are applied
- Write torque equation
- Write torque equation in column vector form
- Force and torque equations are master equations
Find tension force in each rope

**Translation**

Known:
- $d_1 = 2.1 \text{ m}$
- $D_1 = 5.5 \text{ m}$
- $d_2 = 1.5 \text{ m}$
- $D_2 = 7.2 \text{ m}$
- $m = 65 \text{ kg}$

$$\theta_1 = \tan^{-1} \left( \frac{d_1}{D_1} \right) = 20.9^\circ$$

$$\theta_2 = \tan^{-1} \left( \frac{d_2}{D_2} \right) = 11.8^\circ$$
Conceptual Model

Find tension force in each rope

\[
0 = \vec{F}_{T1} + \vec{F}_{T2} + \vec{F}_g + \text{negligible}
\]

\[
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
-F_{T1} \cos \theta_1 \\
0 \\
+F_{T1} \sin \theta_1
\end{bmatrix} +
\begin{bmatrix}
+F_{T2} \cos \theta_2 \\
0 \\
+F_{T2} \sin \theta_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
-mg
\end{bmatrix}
\]

Drag and buoyant forces
Example N5.  

Find magnitude of the tension force in each chain

Translation:  
Known:  
\[ r_T = D = 4.4 \text{ m} \]  
\[ L = 6.6 \text{ m} \]  
\[ \phi = 45^\circ \]  
\[ \theta = 180^\circ - \phi = 135^\circ \]  
\[ m = 580 \text{ kg} \]

Conceptual model:  
\[ \vec{F}_C \] = contact force from hinge
\[ \vec{F}_T \] = tension force exerted by each chain
\[ \vec{F}_g \] = weight of drawbridge

\[ 0 = \vec{r}_C \times \vec{F}_C + \vec{r}_T \times 2\vec{F}_T + \vec{r}_CM \times \vec{F}_g + \text{negligible} \]

\[
\begin{align*}
0 &= \begin{bmatrix} F_{Cx} \\ 0 \\ F_{Cy} \end{bmatrix} + \begin{bmatrix} -2F_T \cos \phi \\ 0 \\ 2F_T \sin \phi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \\
0 &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -2r_T F_T \sin \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} L mg \end{bmatrix} \\

r_C &= 0 \\
\text{Uniform drawbridge} \Rightarrow r_{CM} = \frac{1}{2} L
\end{align*}
\]
Group Problems

- N5S.1
- N5S.6
- N5S.7
- N5S.9