Beams and Slabs

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Beams

• Deflection or sag is important
• With columns, we want to know how much weight they can take before failure
• With beams, the maximum weight is usually less important than the deflection. People will become uncomfortable walking on a sagging floor well before it actually comes close to the breaking point.

Simply supported beam

One end fixed – no horizontal translation or vertical movement
• Other end on a roller to allow for the shortening or lengthening of the beam

Beams and Slabs

• A beam is a linear structural member with loading applied perpendicular to its long axis
• The load on a beam is a bending load
• Bending is the tendency of a member to bow
• A lintel is a term sometimes used for a short beam
• A slab is a 2 dimensional piece with loaded the same way – it is an extension of the beam just as the bearing wall is an extension of the column.
Beam stresses

- Compression at the top of the beam
- Tension at the bottom
- Because compression and tension are occurring in parallel, shear is also present

Shear

- Shear effects are important for beams because their two faces are moving opposite to one another

Shear

- Stack of planks on top of one another will sag a lot more than a single piece of wood

Shear

- Glue or a small piece called a KEY may be used prevent this slippage
Materials

• The best materials for beams are those which have similar strength in tension and compression
• Wood and steel are good materials
• Concrete and masonry are poor choices for long beams
• Lintels are short beams and are often made from concrete

Reinforcement

• Concrete beams are reinforced with steal to prevent tensile cracking
• Steel reinforcing bars are commonly on the lower half of the beam

Prestressed and posttensioned concrete beams

• Steel must begin to stretch before it can offer any bending resistance.
  – This little bit of stretch can result in cracks in the bottom surface of a concrete beam
• Steel is often stretched and installed before the concrete is poured
  – Once the concrete hardens, the steel is released putting the concrete in compression - camber
• Alternately, the steel is placed in hollow sheathes in the concrete and there is no bond between the two

Prestressing process
Posttensioning process

Beam deflection
- Factors that affect beam deflection include
  - Span
  - Depth
  - Width
  - Material
  - Load location
  - Cross-sectional shape
  - Longitudinal shape

Span
- The deflection of a beam increases as the cube of the span
- Doubling the length, increases the sag by a factor of 8

Figure 8.10: The effect of span on deflection. Deflection increases as the cube of the span.
Width

- Deflection varies with its cross-sectional dimensions
- Deflection is inversely proportional to the horizontal dimension
  - Doubling the width reduced the deflection by one half

\[
\text{deflection} \propto \frac{1}{(\Delta \text{width})}
\]

Depth

- Deflection is inversely proportional to vertical depth
  - Doubling the depth, reduces the deflection by a factor of 8

\[
\text{deflection} \propto \frac{1}{(\Delta \text{depth})^2}
\]

- A beam has more stiffness by increasing the depth, as compared to the width

Width and depth

Material strength

- Deflection is inversely proportional to the modulus of elasticity of the material
  - Aluminum will deflect 3 times as much as a comparable steel beam

\[
\text{deflection} \propto \frac{1}{(\Delta \text{modulus of elasticity})}
\]
Load Location

- Deflection at midspan is effected by the location of the load
  - Increases as the load moves toward the middle of the beam

Cross sectional shape

- A problem with beams is the understressing of the material near the center of the cross section
- Distributing the material away from the central axis increases bending resistance
- Box and I-shapes are examples

Longitudinal Beam Shape

Deflection

- If the span or length is increased by a factor 2, the deflection increases by a factor 8 \((2^3)\)
- If the depth increases by a factor of 2, the deflection decreases by a factor of 8 \((2^3)\)
- If the width increases by a factor of 2, the deflection decreases by a factor of 2
Multiple Beams

- To span a space (e.g., bedroom on 2nd floor of a house), we would want multiple, closely spaced beams, known as Joists.

Joists

- Joists are closely spaced beams
- Rectangular space
- Which way should the beams be laid, N-S or E-W?
- Shorter direction results in less sag
- Works well for smaller spaces
- DVD beam slide 24

Beam Grid

- Increase the structural stability by using a Beam Grid
- Interlocking 2 dimensional beams
  - better support than the 1D joists
  - the continuity of the beams is assumed
  - steel can be welded
  - reinforced concrete with bars that bypass each other
  - wood is not suitable
Slabs

- 2D horizontal supports
- Just as a continuous bearing wall is more than just a bunch of columns, a slab is more than just a bunch of beams – the load is distributed over a wider area
- How the slab behaves (how it deflects under load) depends on how it is supported

Types of Slabs

- A one-way slab has supports on two parallel sides, but not the other two
- A two-way slab has supports in both directions (either interlocking beams, or dropped panels) – stronger and more efficient than a one-way slab if the distances in each direction are comparable
- Flat plates just rest on random columns – the slab must be reinforced at the points of support, since there is so much stress there

DVD beams slide 2, 3, 4
Ribbing

- The slab need not be a solid piece
  - removes a lot of needless weight if the middle portion of the slab (which experiences little stress) can be removed.
- Leads to "ribbed" slabs
  - most of the concrete is at the top of the slab,
  - more “ribs” with reinforcing steel are placed at the bottom to take the tension.
  - Thus one has a slab on top of joists (i.e. slab on top of beams or a beam grid).

Isostatic Ribs

- Usually the ribs are laid out in convenient, rectangular forms
- In principle it is more efficient to use isostatic ribs that follow the lines of the bending stresses and so prevent shear
- Expensive!

[Images of ribbed slabs and isostatic ribs]
Cantilever and Continuous Beams

- So far we've considered simply supported beams (or slabs) that just rest on one support at each end
- We get slightly different conditions if a beam is actually clamped at each end, or if it rests on multiple supports
- A beam that is clamped at each end is a Cantilever
- A beam that rests on multiple supports is a Continuous Beam

Cantilever

- A cantilever which is fixed at one end and loaded perpendicular to its axis
- A cantilever can be vertical or horizontal
- A flagpole is an example of a cantilever
- Assuming the latter, how does this change in attachment affect the beam?

Cantilever deflection

- LOWER portion is in compression
- UPPER portion is in tension
- exactly the opposite of the simply supported beam
- DVD beams slide 18

Cantilever deflection

- The sag of a cantilever again depends on span, width, depth, and material
  - just treat it as half a simply-supported beam for these cases
  - more material away from the axis is good
  - I-beam or hollow tube are good choices for a cantilever
  - DVD Beams slide 5
Cantilever deflection – con’t

• The sag also depends on the shape of the cantilever
• Unlike a simply-supported beam, the cantilever has the greatest stress near the clamped end, so we want the most material at that end and little material at the free end
• A tapered shape is most efficient — need depth near support, but not out near load

Falling Water

• The most famous cantilevered structure
• a house built for the Kauffman family
• built by the famous architect Frank Lloyd Wright
Overhanging Beam

• In contrast to a cantilever is an overhanging beam
• As the name implies, the overhanging beam is a beam that extends out beyond its last support
• Unlike a cantilever the overhanging beam is allowed to rotate instead of being held rigidly
• DVD beam slide 5

• Which case would lead to more deflection?
Continuous Beams

• Some projects require a large open area, unbroken by supporting columns
• But for many projects it is acceptable (and much more efficient) to space columns across a space

Continuous beams

• In this case, we could place individual short beams that extend from one column to the next
• Or we could use one long beam that stretched across all the columns

• Are these two spans equivalent?

Continuous beams

• NO, the continuous beam will continue to affect portions beyond the closest support
• Instead of only a convex shape between each column, we now get a concave shape over the supports
• DVD beam slide 12
• Just as the load bends the beam down, the support bends it up
• That means that the length of stressed beam between each set of supports is shortened, and hence the deflection is less!
• Or, put another way, for the same amount of deflection, we could use a narrower beam, saving material and hence cost
Gerber Beam

• Alternately, instead of a single long piece, we CAN break the beam down into shorter segments and STILL get the reduced deflection of a continuous beam. How?
• The key is the inflection points of the continuous beam
• We can’t sever the beam at the supports because the stress is high there.
• But at the ¼ and ¾ points where the beam changes from concave up to concave down, there is NO stress.
• Thus at THOSE points it is OK to break the longer segment and replace it with shorter beams joined by pinned connections
• Such an arrangement is called a Gerber beam

Inflection Points

Forces in a Cantilever Bridge

Tension

Compression
Sir Benjamin Baker's demo