Interactions Transfer Momentum

Chapter Overview

Introduction
This chapter opens the subdivision on conservation of momentum. In this chapter, we will apply what we learned in chapter C2 about vectors to explore fundamental principles and concepts at the very heart of mechanics.

Section C3.1: Velocity
A particle is a hypothetical object whose position is a mathematical point. In what follows, I will describe concepts in terms of particles, but will apply them to macroscopic objects as well. (We will see why we can do this in chapter C4.)

We define a particle's velocity \( \vec{v} \) at a given instant of time in this way:

\[
\vec{v} = \frac{d\vec{r}}{dt} \quad \text{or} \quad \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix}
\]

(C3.2)

**Purpose:** This equation defines a particle's velocity vector \( \vec{v} \).

**Symbols:** \( d\vec{r} \) is the particle's displacement during the time interval \( dt \).

**Limitations:** The interval \( dt \) must be so short that \( \vec{v} \approx \) constant during \( dt \).

**Notes:** Speed \( v = \text{mag}(\vec{v}) \).

Section C3.2: Interactions Transfer Momentum

All known interactions obey the following principle:

The momentum-transfer principle: Any interaction between two objects affects their motion by transferring momentum from one to the other.

Experimentally, we find that this works if we define a particle's momentum to be

\[
\vec{p} = m\vec{v}
\]

(C3.4)

**Purpose:** This equation defines a particle's momentum vector \( \vec{p} \).

**Symbols:** \( m \) is the particle's mass, \( \vec{v} \) is its velocity.

**Limitations:** This equation applies to particles only (but see chapter C4).

Gaining an amount of momentum in one direction is the same as losing the same amount of momentum in the opposite direction, so we can consider either particle to be the source of the momentum transfer (to switch donors, switch signs).

Section C3.3: Impulse and Force
We call the amount of momentum \( d\vec{p} \) that a specific interaction \( A \) contributes to a given particle's total momentum during a short time interval \( dt \) the impulse delivered by that interaction during \( dt \). Experiments show that
\[ \begin{align*}
\Delta p &= [d\vec{p}]_A + [d\vec{p}]_B + \cdots \\
\text{Purpose:} & \quad \text{This equation describes how multiple interactions affect a particle's motion.}
\end{align*} \]

Symbols: \([d\vec{p}]_A, [d\vec{p}]_B, \ldots\) are the impulses contributed to the particle by interactions A, B, etc. during a given short time interval \(dt\), and \(d\vec{p}\) is the resulting change in the particle's actual momentum \(\vec{p}\) during \(dt\).

Limitations: This equation applies to particles only (but see chapter C4).

Note: The brackets in the notation \([d\vec{p}]_A\) distinguish the small amount of momentum that a specific interaction \(A\) contributes to a particle during \(dt\) from the net change \(d\vec{p}\) in the particle's actual momentum during that interval.

We define the force that an interaction between objects exerts on either object to be the rate at which the interaction delivers impulse to the object:

\[ \vec{F}_A = \frac{[d\vec{p}]_A}{dt} \]

Purpose: This defines the force \(\vec{F}_A\) that interaction \(A\) exerts on a particle.

Symbols: \([d\vec{p}]_A\) is the impulse delivered by the interaction to the particle during the small time interval \(dt\).

Limitations: The interval \(dt\) should be small enough that \(\vec{F}_A\) is constant during \(dt\). (If it is not, \(\vec{F}_A\) becomes the average force during the interval.)

Notes: The SI unit of force is the newton, where 1 N = 1 kg m/s². (The English unit is the pound, where 1 lb = 4.45 N.) Force \(\vec{F}_A\) has the same direction as the impulse \([d\vec{p}]_A\) being delivered.

Since the momentum that an interaction between two particles contributes to one must come from the other, the forces that a given interaction exerts on two particles must be equal in magnitude but opposite in direction (this is Newton's third law).

**Section C3.4: Mass and Weight**

In physics, mass and weight are completely distinct concepts. A particle's mass expresses how much impulse is required to produce a certain change in the particle's velocity. An object's weight \(\vec{F}_w\) is the total force that gravitational interactions exerts on the object at a given point. Weight is a vector measured in newtons, while mass is a scalar measured in kilograms. Experimentally, though,

\[ \vec{F}_w = mg \]

Purpose: This equation describes how weight is related to mass.

Symbols: \(\vec{F}_w\) is the weight acting on an object; \(m\) is the object's mass; \(g\) is the gravitational field vector at a certain point in space.

Limitations: There are none (until we study general relativity, at least).

Notes: Near the earth's surface, \(g\) points toward the earth's center, and the gravitational field strength \(g = \text{mag}(g) = 9.8\ \text{N/kg}\).

**Section C3.5: Momentum Flow and Motion**

One can use momentum-transfer ideas to make qualitative predictions about a particle's motion. See the section for details.

**Section C3.6: Physics Skills: Illegal Vector Equations**

This section discusses errors that beginners often make in dealing with vectors.
C3.1 Velocity

A particle is a hypothetical object having zero volume, and thus its position is a mathematical point in space. We saw in chapter C1 that this idealization is an excellent model for an elementary particle. However, we will see in chapter C4 that it also proves to be a useful model for macroscopic objects.

In the remainder of this chapter, I am going to talk about various quantities and principles as if they applied only to particles, because particles are so well defined mathematically. However, I will freely apply these quantities and principles to macroscopic objects in examples and arguments. We will see why we can get away with this in chapter C4.

In chapter C2, we defined the position \( \vec{r} \) of a point in space to be its displacement from the reference frame origin to the point in question (see figure C3.1a). We also saw that we can consider the displacement \( \Delta \vec{r} \) between two arbitrary points to be the difference of their positions:

\[
\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 \quad \text{or} \quad 
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z 
\end{bmatrix} = 
\begin{bmatrix}
x_2 \\
y_2 \\
z_2 
\end{bmatrix} - 
\begin{bmatrix}
x_1 \\
y_1 \\
z_1 
\end{bmatrix} = 
\begin{bmatrix}
x_2 - x_1 \\
y_2 - y_1 \\
z_2 - z_1 
\end{bmatrix} \tag{C3.1}
\]

(see figure C3.1b). Since at every instant of time a particle occupies a mathematical point in space, we can describe its motion as a sequence of displacements at various instants of time.

In chapter C1, we qualitatively defined a particle’s velocity \( \vec{v} \) at a given instant of time \( t \) to be a vector whose magnitude is the particle’s speed \( v \) and whose direction is the particle’s direction of motion at that instant of time. We are now in a position to define \( \vec{v} \) more quantitatively. Let \( dt \) be the duration of a time interval containing the instant \( t \) at which we want to know \( \vec{v} \), and let \( d\vec{r} \) be the particle’s tiny displacement during that tiny time interval. If \( dt \) is sufficiently short that neither the particle’s speed nor its direction of motion changes appreciably during that time interval, then the particle’s velocity is \( \vec{v} = d\vec{r}/dt \) (that is, the displacement vector \( d\vec{r} \) multiplied by the scalar \( 1/dt \)):

\[
\vec{v} = \frac{d\vec{r}}{dt} \quad \text{or} \quad 
\begin{bmatrix}
v_x \\
v_y \\
v_z 
\end{bmatrix} = \frac{1}{dt} 
\begin{bmatrix}
dx \\
dx \\
dx 
\end{bmatrix} = \frac{dx}{dt} \quad \frac{dy}{dt} \quad \frac{dz}{dt} \tag{C3.2}
\]

**Purpose:** This equation defines a particle’s vector velocity \( \vec{v} \).

**Symbols:** \( d\vec{r} \) is the particle’s displacement during the time interval \( dt \).

**Limitations:** The interval \( dt \) must be short enough that \( \vec{v} \approx \) constant during \( dt \).

**Notes:** The direction of \( \vec{v} \) is the direction of motion; \( \text{mag}(\vec{v}) = \) speed \( v \).

Does this definition coincide with our earlier qualitative definition? Consider the direction aspect first. Since \( 1/dt \) is a positive scalar, the vector \( \vec{v} \) ends up having the same direction as the short displacement \( d\vec{r} \), which in turn accurately reflects the direction that the object is moving if and only if \( dt \) is sufficiently short, as shown in figure C3.2 (this is why this limitation is important). As for the magnitude, you probably learned in junior high school that an object’s speed is the distance traveled in a time interval divided by
that time interval. According to equation C3.2, the particle’s speed is

\[ v = \text{mag}(\vec{v}) = \sqrt{v_x^2 + v_y^2 + v_z^2} \]

\[ = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \]

\[ = \sqrt{\frac{dx^2 + dy^2 + dz^2}{dt}} \]  \hspace{1cm} (C3.3)

where \( |dr| = \text{mag}(\vec{v}) \) is the distance the particle travels during the time interval \( dt \). So equation C3.3 is consistent with the familiar definition of speed.

**Exercise C3X.1**

A particle moves with a constant speed around a circle. Is its velocity constant?

**Exercise C3X.2**

During a “sufficiently short” time interval of 0.25 s, a hypothetical particle moves a displacement of 4.0 m east, 1.0 m south, and 0.1 m downward relative to the earth’s surface. What are the components of this particle’s velocity at this time in a reference frame in standard orientation on the earth’s surface? What is the particle’s speed?
C3.2 Interactions Transfer Momentum

In chapter C1, we defined an interaction between two objects to be a physical relationship between them that allows each to affect the other's motion. We are finally in a position to construct a model that describes more precisely what happens during an interaction.

Consider the experiment shown in figure C3.3, where a cart moving with a certain initial speed \( v_0 \) collides with an identical cart at rest. If you actually do the experiment (assuming that the effects of friction are small and the carts' bumpers are sufficiently "springy"), you will find that the collision interaction affects the carts' motion by bringing the first to rest and sending the second cart away with the same speed the first had initially. Qualitatively it looks as if the contact interaction between the carts during the collision causes the first cart's motion to be entirely transferred to the second cart! Is this what always happens in a collision?

We can check this by doing the experiment shown in figure C3.4, which is just like the collision shown in figure C3.3 except that the initially moving cart is now twice as massive as before. If you do this experiment (assuming low friction and springy bumpers), you will find that the initially moving cart is not brought to rest by the collision interaction, but rather continues to move forward with about one-third of its original speed \( v_0 \) while the lighter cart moves away from the collision with a speed of about \( \frac{2}{3}v_0 \).

This experiment scuttles the idea that one cart's motion is entirely transferred to the other cart. Indeed, in this case we seem to have more "motion" after the collision than we did before, since the total of the carts' speeds before the collision was \( v_0 \) and \( \frac{2}{3}v_0 \) afterward. On the other hand, the front cart still gains motion, and the other loses it in the collision. Let us see if we can save the basic concept that some quantity related to motion is being transferred from the initially moving cart to the one initially at rest.

Since the only change we made was in the mass of the moving cart, perhaps the "quantity related to motion" has something to do with mass. Indeed, you may know from painful experience that the effects you suffer when an object moving at a given speed hits you worsen as its mass increases (would you rather be hit by a tennis ball moving at 20 mi/h or a truck moving at the same speed?). On the other hand, your suffering increases with the object's speed, too! So perhaps the "quantity of motion" we are interested in here is the product of the object's mass and speed. Let us see where this hypothesis gets us.

**Figure C3.3**
An example collision between identical carts. (The final speeds are the actual experimental results in this situation.)

**Figure C3.4**
An example collision where one cart is twice as massive as the other. (The final speeds are the actual experimental results in this situation.)
C.3.2 Interactions Transfer Momentum

Figure C.3.5
(a), (b) A collision between two carts where the right cart is twice as massive as the left cart. (c) Note that the collision interaction transfers a "quantity of motion" (assumed here to be a vector) equal to \( \frac{1}{3} m_0 \) rightward from the left cart to the right cart.

Let us take the mass of the lighter cart to be \( m_0 \), the mass of the initially moving cart is then \( 2m_0 \). Our hypothetical "quantity of motion" for the initially moving cart is thus \( 2m_0 \). The collision gives the right cart a "quantity of motion" of \( m(\frac{1}{2}v_0) = \frac{1}{3} m_0 \). If we subtract this from \( 2m_0 \), we get \( 2m_0 - \frac{1}{3} m_0 \), which is indeed the mass of the left cart \( 2m_0 \) times its observed speed of \( \frac{1}{2}v_0 \).

So this collision interaction does indeed seem to transfer a "quantity of motion" equal to \( \frac{1}{3} m_0 \) from the back cart to the right cart, with some remaining in the left cart. The explanation works for the initial experiment as well, where the first cart transferred to the second cart a "quantity of motion" equal to \( m_0 \) (although it is curious that the magnitude of the "quantity of motion" transferred is different in the two experiments).

Flush from this success, let us now try an experiment where we reverse the masses of the carts. The results of such an experiment (as you can verify) are shown in figure C.3.5. After the collision, the massive right cart moves ahead with a speed of \( \frac{1}{4}v_0 \), and the lighter left cart rebounds backward with a speed of \( \frac{1}{4}v_0 \). These results are catastrophic for our model. Our "quantity of motion" for the initially moving cart is \( m_0 \), but its value for the right cart after the collision is \( 2m(\frac{1}{4}v_0) = \frac{1}{2}m_0 \). Again, we seem to end up with more "quantity of motion" than we had before.

But wait! What if direction matters? What if our "quantity of motion" in fact should be a vector with a direction parallel to the direction of motion? Then the initial quantity of motion is an arrow pointing forward with a length proportional to \( m_0 \). The interaction gives the right cart a quantity of motion equivalent to a rightward arrow of length \( \frac{1}{4} m_0 \). To find out what this gift does to the first cart, we have to subtract this arrow from the first cart's initial arrow, which is the same as adding a leftward arrow of length \( \frac{1}{4} m_0 \). The result (as shown in figure C.3.5c) is that the final quantity of motion for the left cart is \( \frac{1}{4} m_0 \) in the leftward direction, consistent with what is observed.

Physicists now call the vector quantity that we have discovered momentum (although "quantity of motion" is in fact the English translation of the Latin term that Newton used). We formally define a particle's momentum to be:

\[
\vec{p} = m\vec{v} \quad (C.3.4)
\]

- **Purpose:** This equation defines a particle's momentum vector \( \vec{p} \).
- **Symbols:** \( m \) is the particle's mass; \( \vec{v} \) is its velocity.
- **Limitations:** This equation applies to particles only (but see chapter C4), and only in the context of newtonian mechanics.
- **Notes:** The SI units for \( \vec{p} \) are kilogram-meters per second (kg·m/s).

A vector model of momentum

The definition of the momentum of a particle
You can easily verify that the vector model of momentum transfer we developed for the experiment shown in figure C3.5 also works for the experiments shown in figures C3.4 and C3.3, as well as for collisions where the carts have arbitrary relative masses, where bumpers that are not springy or involve interactions that are not contact interactions (e.g., magnetic repulsion), or even where both carts move initially. Our model does not predict the amount of momentum transferred in the collision (that depends somewhat on the type of interaction—see chapter C12), but still tells us something very important about interactions.

Exercise C3X.3

Imagine that a cart with mass \( m \) moving with speed \( v_0 \) collides with an identical cart at rest. After the collision, the carts stick together and move together at a speed of \( \frac{1}{2} v_0 \) in the same direction as the originally moving cart. Show (by drawing a picture analogous to figure C3.5c) that we can also think of this collision as transferring momentum from the left cart to the right cart. Compute the magnitude and direction of that momentum transfer.

We are encouraged by this evidence to generalize boldly:

The momentum-transfer principle: Any interaction between two objects affects their motion by transferring momentum from one to the other.

The model of motion that this principle expresses has proved over the centuries to be an extremely powerful and far-reaching model for understanding interactions, and (with some refinements) it applies even in our most modern theories. In fact, you may recall from chapter C1 that in quantum field theory, we model an interaction between two particles by imagining that they exchange the interaction’s mediator particle; these mediators quite literally carry momentum between the interacting particles! This model is the very heart of modern mechanics, and many of the ideas in this unit (and essentially everything in unit N) follow as logical consequences of this model.

The rest of this chapter and chapter C4 are devoted to giving you just an overview of these consequences.

By the way, it does not matter in this model which of the two interacting objects we consider to be the donor and which the recipient of the momentum transferred by the interaction. In the collision shown in figure C3.5, either we can think of the left cart as giving the right cart \( \frac{1}{2} m_0 \) of rightward momentum and losing \( \frac{1}{2} m_0 \) of rightward momentum as a result, or we can think of the right cart as giving \( \frac{1}{2} m_0 \) of leftward momentum to the left cart and losing \( \frac{1}{2} m_0 \) of leftward momentum as a result. The definition of the vector inverse ensures that the results are the same either way. This is good, because it would be strange if the fundamental model of the interactions between two objects treated the two objects asymmetrically.

C3.3 Impulse and Force

Physicists call the amount of momentum that a particular interaction \( A \) between two particles transfers to either particle during a short interval of time the impulse \([d\mathcal{P}]_A\) that the interaction delivers to that particle during that interval. For example, the impulse delivered to the right cart by the collision interaction \( C \) shown in figure C3.4 is \([d\mathcal{P}]_C = \frac{1}{2} m_0 \) right.
A financial analogy may help us understand some subtle distinctions in meaning between the terms a particle's momentum \( \vec{p} = m \vec{v} \), impulse, and momentum in general. Think of the particle's momentum \( \vec{p} = m \vec{v} \) as being like a person's net financial worth. An interaction is like a financial transaction that increases one person's net worth at the expense of the other's. An impulse is like a check that a person writes or receives in the transaction. The term momentum is, like money, a general term for both what each particle (person) has and what is being transferred. (The only flaw in this analogy is that momentum is a vector quantity, while money is a scalar.)

If a particle participates in only a single interaction (call it interaction \( A \)) with one other particle, then the total change in its momentum due to the interaction during a short interval of time will be equal to the impulse that it receives from the interaction during that time: \( d\vec{p} = [d\vec{p}]_A \). But what happens if a particle participates in multiple interactions with multiple partners? The financial analogy suggests the answer. During a given month, a person may write and/or receive checks involving many other people. The change in the person’s net worth during that month is just the arithmetic sum of the check amounts (treating incoming checks as positive numbers and outgoing checks as negative numbers). By analogy, the change \( d\vec{p} \) in the momentum of a particle that participates in multiple interactions during a given interval is the vector sum of the impulses it receives during that interval:

\[
d\vec{p} = [d\vec{p}]_A + [d\vec{p}]_B + \cdots
\]

(C3.5)

**Purpose:** This equation describes how multiple interactions affect a particle’s motion.

**Symbols:** \([d\vec{p}]_A, [d\vec{p}]_B, \ldots\) are the impulses (momentum contributions) delivered to the particle by interactions \( A, B, \ldots \) during a given short time interval \( dt \), and \( d\vec{p} \) is the resulting change in the particle’s actual momentum \( \vec{p} \) during that time interval.

**Limitations:** This equation applies to particles only (but see chapter C4).

In spite of the analogy, it is not obvious that impulses should really add this way, but 300 years of experiments support this simple model.

Please note in the preceding paragraphs that I am using brackets to distinguish the amount of momentum \([d\vec{p}]_A\) (i.e., impulse) that a given interaction \( A \) contributes to a particle from the actual change \( d\vec{p} \) in the value of that particle’s momentum \( \vec{p} = m\vec{v} \). As equation C3.5 makes clear, these quantities are not equal in general: the net change \( d\vec{p} \) in the particle’s momentum during a given time interval \( dt \) is determined by the vector sum of all of the impulses that various interactions contribute during that interval. I believe that recognizing this distinction is very important if one is to develop a clear understanding of the momentum-transfer model, so I will use this notation carefully and deliberately in what follows, and I recommend that you do likewise. Read \([d\vec{p}]_A\) as meaning “the contribution that interaction \( A \) makes to \( d\vec{p} \).”

The term impulse fairly intuitively expresses what a collision interaction does to a particle’s motion, but you can even think of a continuing interaction as delivering a continuous series of very tiny impulses to a particle, which you can think of as little “taps” in a certain direction. For example, you perhaps know that the speed of a dropped object increases as the object falls. We can see why this is so if we imagine the gravitational interaction between the object and the earth as delivering a continuous series of tiny downward taps.
to the object, each one increasing the object’s downward momentum by a tiny amount. As a result, the falling object accumulates downward momentum (and thus downward speed) as time passes.

In the limit as the taps become extremely tiny and the time between them becomes infinitesimal, the momentum flow into the particle becomes continuous, almost like a fluid accumulating in the particle. This “momentum flow” image is very useful for interactions that, unlike collisions, last a significant amount of time.

We define the force $\mathbf{F}$ that an interaction exerts on a given object to be the rate at which momentum flows into the object because of that interaction:

$$\mathbf{F}_A = \frac{\mathbf{d}[\mathbf{p}]}{\mathbf{d}t} \quad (C.3.6)$$

**Purpose:** This equation defines the force $\mathbf{F}_A$ that interaction $A$ exerts on a particle.

**Symbols:** $\mathbf{d}[\mathbf{p}]$ is the small impulse the interaction delivers to the particle during the small time interval $\mathbf{d}t$.

**Limitations:** The interval $\mathbf{d}t$ must be small enough that $\mathbf{F}_A$ is constant during $\mathbf{d}t$.

**Notes:** The SI unit of force is the newton, where $1 \text{ N} = 1 \text{ kg \cdot m/s}^2$. Force $\mathbf{F}_A$ has the same direction as the impulse $\mathbf{d}[\mathbf{p}]$ being delivered.

The first note in the box acknowledges that force is such a useful concept in Newtonian mechanics that it has its own special SI unit. Since $\mathbf{d}[\mathbf{p}]$ must have the same units as momentum (kilogram-meters per second, the units of mass times velocity), consistency requires that the units of force be those of momentum/time, or kilogram-meters per second, per second (kg·m/s)/s — kilogram-meters per second squared (kg·m/s²). We define one newton to be the force experienced by a particle receiving an impulse of one kilogram-meter per second every second (1 N = 1 kg·m/s²). A newton is very roughly the magnitude of the upward push required to hold a music CD (in its case) at rest in the earth’s gravitational field.

How well does this definition of force coincide with the colloquial idea of force as a push or pull? Consider an object floating at rest that is not interacting with anything. If you push on this object in a certain direction for a certain time interval $\mathbf{d}t$, the contact interaction between your hand and the object will give it an impulse $\mathbf{d}[\mathbf{p}] = \mathbf{F}_C \mathbf{d}t$ in the same direction, and if it was initially at rest, it will end up moving in that direction. A small $\mathbf{F}_C$ delivers a small impulse during $\mathbf{d}t$, so the object’s change in velocity will be small, as we would expect from a weak push. On the other hand, a large $\mathbf{F}_C$ gives it a proportionally larger impulse, prompting it to respond more vigorously, just as we would expect from a stronger push. Thus our technical definition does seem to reproduce what we would expect from the “force = push” concept, while also providing a quantitative definition of a force’s strength and its exact effect on a particle’s motion.

**Exercise C3X.4**

Consider the collision between carts shown in figure C3.5. Imagine that $m = 1.0 \text{ kg}$, $v_0 = 1.0 \text{ m/s}$, and the contact interaction lasts for $0.05 \text{ s}$. Assume that the force that the contact interaction exerts on the front cart is constant (this is not really likely, but let us pretend). What is the magnitude of this force?
(Note: When we compute the force by pretending it is constant over a certain interval when it really probably is not, we signal this by saying that we are computing the average force over the time interval.)

We can visualize an interaction between two particles as a being like a hose that carries a flow of momentum from one to the other. Since any momentum that flows out of one must flow into the other, the magnitude of the rate of momentum flow must be the same for both particles. This statement and the definition of force directly imply that

A given interaction between particles A and B must exert a force on B that is equal in magnitude and opposite in direction to the force that is exerts on A: \( F_{A\to B} = -F_{B\to A} \).

(The minus sign is there because any momentum that flows into B has flowed out of A.) For example, figure C.3.a in section C.3.2 shows that in the collision shown, the impulses delivered to the two carts (and thus the forces) are indeed equal in magnitude and opposite in direction. Physicists call this statement *Newton's third law*; we will study it in greater detail in unit N.

**C.3.4 Mass and Weight**

Equation C.3.4 defines momentum in terms of mass and velocity. While I carefully defined velocity in equation C.3.2 in terms of more basic ideas, I really have not yet defined what I mean by mass, trusting instead that you intuitively knew what I meant. I would like to deal with this issue more carefully now.

The principle of momentum transfer is the most fundamental physical idea in this chapter. We in fact discovered the definition \( \vec{p} = m \vec{v} \) by trying to make this principle work experimentally. The point is that we can define momentum independently of mass as being whatever "quantity of motion" an interaction transfers.

Viewed from this perspective, a particle’s mass simply expresses the relationship between its momentum and its velocity, each of which we can define separately. Qualitatively, a particle’s mass expresses how much impulse we need in order to change its velocity by a given amount. Conversely, the greater a particle’s mass, the smaller its velocity change will be for a given impulse: a particle’s mass expresses the degree to which it resists changes in its motion.

An operational definition defines a physical quantity by describing a procedure for measuring that quantity. (Operational definitions are useful in physics because they firmly anchor quantities in empirical reality, avoiding the potential vagueness of verbal definitions.) An operational definition of mass based on the principle of momentum transfer goes as follows. Imagine we have particle of known mass \( m_1 \) and a particle whose mass \( m_2 \) we would like to measure. We set things up so that the two particles participate in a repulsive interaction with each other (perhaps by placing a compressed spring between them) but do not interact significantly with anything else. We hold them initially at rest, and then we release them so that the repulsive interaction, acting alone, drives them apart (see figure C.3.6). We then measure the particles’ speeds \( v_1 \) and \( v_2 \) after some given interval of time. (Note that each speed is equal to the magnitude of that particle’s change in velocity, since the initial velocity of each was zero.)

The principle of momentum transfer means that each particle must get the same magnitude of impulse. Therefore, if we want mass to express the particle’s resistance to changes in its velocity for a given impulse, then the
The SI unit of mass

Mass and weight are completely distinct concepts!

The relationship between mass and weight

\[ F_w = mg \]  

(C3.8)

**Purpose:** This equation describes the relationship between weight and mass. 

**Symbols:** \( F_w \) is the weight acting on an object, \( m \) is the object's mass; \( g \) is the gravitational field vector at a certain point in space.

**Limitations:** There are none (until we study general relativity, at least).

**Notes:** Near the earth’s surface, \( g \) points toward the earth’s center, and the gravitational field strength \( g = \text{mag}(g) = 9.8 \text{ N/kg} \).

Surprisingly, the gravitational field vector \( g \) in equation C3.8, while it does depend on one's exact position relative to a strongly gravitating object such as the earth, is completely independent of the nature of the object involved, meaning that \( \text{mag}(F_w) \) is strictly proportional to \( m \) for all objects at a given point in space (a fact that has no explanation in Newton’s mechanics, but is explained by general relativity). Within any region of space where \( \text{mag}(g) \) is constant (such as points relatively near the surface of the earth), we consider the magnitude of the weight force acting on an object to be proportional to its mass for all practical purposes.

Note that the **pound** is an English system unit of force (1 lb = 4.45 N), not mass! A 1-kg object experiences a weight force of about 2.2 lb near the earth’s surface.

**Exercise C3X.5**

In an experiment of the type shown in figure C3.6, an object whose mass is known to be 0.50 kg has a final speed of 3.0 m/s, while the other object has a final speed of 2.0 m/s. What is its mass?
C3.5  Momentum Flow and Motion

One can fairly easily use either impulse or momentum flow ideas to make qualitative predictions about an object's motion. Consider, for example, throwing a ball so that its initial velocity has both an upward and a forward component, and imagine that after it is launched, the ball interacts only gravitationally with the earth (friction interactions are negligible). Figure C3.7 shows how we can predict the trajectory by treating the gravitational force on the ball as a series of small downward taps separated by a given time interval $dt$. The ball's displacement between taps will be parallel to its momentum $\vec{p}$ after the last tap (and proportional in length to $\vec{p}$, because the ball's displacement is proportional to its velocity between the taps, which in turn is proportional to $\vec{p}$). Each tap, however, changes the ball's momentum so that its new momentum (black arrow) is the vector sum of its old momentum (gray arrow) and the downward impulse (colored arrow) delivered by the tap (since $d\vec{p} = \vec{F}_{\text{impact}} dt = d\vec{p}_{\text{impact}}$). The ball subsequently moves in the direction indicated by the new momentum until the next tap. Figure C3.7 shows the resulting trajectory. Perhaps you can imagine that in the limit that the taps become very small and the time between taps becomes small, the trajectory will become a smooth parabola.

Another whole approach (which I actually find easier to visualize) is to imagine each component of the ball's momentum to be like a graduated cylinder reservoir containing a certain amount of fluid. If the fluid level in the reservoir corresponding to, say, the momentum $x$ component is above the zero mark, the ball's $x$-momentum is positive; if below, the $x$-momentum is negative. Pouring fluid into this reservoir thus increases the component of the ball's momentum in the $+x$ direction, while draining momentum from it decreases that component.

**Figure C3.7**
How to predict the trajectory of a thrown ball by modeling the effect of the gravitational interaction as a series of small taps. The gray arrows represent the ball's momentum before each tap, the colored arrows the impulse delivered by that tap, and the black arrows the ball's momentum after each tap. The inset multiflash picture shows that the actual trajectory of a thrown ball is consistent with this prediction.
In the particular case of the thrown ball, the force exerted by the ball’s gravitational interaction with the earth adds downward momentum to the ball at a constant rate. This is equivalent to draining the reservoir corresponding to the $+z$ component of the ball’s momentum at a constant rate. So the fluid level in this $z$ reservoir steadily drops, eventually passing the zero and continuing into negative territory. On the other hand, since the impulse delivered by the gravitational interaction is purely downward, it has no component in either the $x$ or $y$ direction, so the corresponding $x$ and $y$ reservoirs get neither drained nor filled. By imagining what happens to the levels in these reservoirs as time passes, one can sketch the ball’s momentum vectors at successive instants of time. This allows one to sketch the ball’s trajectory qualitatively, as shown in figure C3.8.

These models for understanding motion have their own strengths and weaknesses. The multiper model can yield a fairly precise prediction of the trajectory, but is somewhat abstract and does not work well for continuously acting forces if you use too few taps. The three-reservoir model is more concrete, but is good for only qualitative predictions. Note also that the model is not completely realistic: one can add to or drain an object’s momentum indefinitely without either overflowing or emptying the “reservoirs”! Use whichever model you understand better and/or better fits the context.

Consider now a cart rolling forward in the $+x$ direction along a level track with nonnegligible friction. This case is more complicated than the case of the ball because the cart participates in multiple interactions. We can usefully think of the cart as participating in three interactions: a gravitational interaction with the earth, a compression contact interaction with the track that prevents it from sinking into the track, and a friction contact interaction with the track that seeks to oppose its motion relative to the track.

In this case, I find the reservoir model most natural. Note that the cart’s vertical momentum is always zero, so since gravity drains the cart’s $z$ reservoir at a constant rate, the compression contact interaction must fill it at the same rate. The impulses delivered by these interactions are entirely vertical, so they leave the $x$ and $y$ reservoirs alone. The friction contact interaction, though, opposes the cart’s motion relative to the track, so transfers momentum in the $-x$ direction to the cart, which is equivalent to draining the cart’s $x$ reservoir. Thus the cart’s $x$-momentum (and thus forward velocity)
decreases steadily until the cart stops. At that point, there is no more relative motion for the friction interaction to resist, so it stops draining the x reservoir, and the cart subsequently remains at rest. Figure C3.9 illustrates the situation as the cart is slowing down.

For our purposes in this unit, a qualitative understanding of the effects of interaction mechanics at the level of these examples is sufficient. We will discuss the effects of interactions on motion in detail in unit N.

Exercise C3X.7
Imagine a pool ball that is initially rolling due north on a pool table. Your opponent, in a blatantly illegal attempt to foil your great shot, blows constantly directly eastward on the ball. Model the ball as a particle that is moving initially northward but experiences a constant eastward force. Sketch momentum arrows for the ball at successive times during your friend’s blowing, using both the multitap and graduated-reservoir models.

C3.6 Physics Skills: Illegal Vector Equations

Since so many important physical quantities are described by vectors, we will be doing a lot of mathematics with vectors from this chapter onward. Doing algebra with vector quantities is a lot like doing algebra with ordinary scalars except in a few important cases. Here is a list of things to keep in mind when doing vector algebra that will help you avoid making common mistakes.

1. Do not set a vector equal to a scalar. A vector quantity has both a magnitude and an associated direction, while a scalar quantity has only a numerical value. Therefore, a vector is never equivalent to a scalar (even if their magnitudes are the same). So avoid writing equations like \( \vec{v} = 20 \text{ m/s} \) when you really mean is \( \text{mag}(\vec{v}) = 20 \text{ m/s} \) or perhaps \( \vec{v} = 20 \text{ m/s} \) south or \( \vec{v} = (20 \text{ m/s}) \hat{j} \). (The only exception is that an equation like \( \vec{v} = 0 \) is acceptable: We conventionally take this to mean that all the components of \( \vec{v} \) are zero.)

2. Vector magnitudes are always positive (by definition!). For example, saying that \( \vec{v} = -32 \text{ m/s} \) is absurd: a particle’s speed is the magnitude of its velocity vector and so must be positive. (If you are tempted to write this, you probably mean something like \( v_x = -32 \text{ m/s} \): a vector component can be negative.)

3. Division by a vector is not defined. If you are given an equation like \( \vec{v} = \frac{d\vec{r}}{dt} \) and are asked to find \( d\vec{r}/dt \), you might be tempted to rearrange it in the usual way to get \( d\vec{r} = \vec{v} \cdot dt \). But the latter equation is meaningless, since division by a vector is not defined: you cannot divide both sides of an equation by a vector. (The right way to solve the vector equation \( \vec{v} = \frac{d\vec{r}}{dt} \) for \( d\vec{r}/dt \) is to take the magnitude of both sides, to get \( \text{mag}(\vec{v}) = \text{mag}(d\vec{r})/dt \), and then to rearrange these scalar values in the usual way to get \( d\vec{r} = \text{mag}(d\vec{r})/\text{mag}(\vec{v}) \).]

4. Remember that \( \text{mag}(\vec{a}_1 + \vec{a}_2) \neq \text{mag}(\vec{a}_1) + \text{mag}(\vec{a}_2) \). See figure C3.30 for an example. To take an even more extreme example, note that if \( \vec{a}_1 \) and \( \vec{a}_2 \) have equal magnitudes but opposite directions, \( \text{mag}(\vec{a}_1) + \text{mag}(\vec{a}_2) = 0 \), but \( \text{mag}(\vec{a}_1 + \vec{a}_2) = 0 \). Similarly, \( \text{mag}(\vec{a}_1 - \vec{a}_2) \neq \text{mag}(\vec{a}_1) - \text{mag}(\vec{a}_2) \). Therefore, whenever you take the magnitude of both sides of a vector equation, check carefully that you do not break up the magnitude of a vector sum (or difference) into the sum (or difference) of magnitudes.

Figure C3.9
The three-reservoir model applied to the moving cart problem at one instant of time. The level in the z reservoir never changes from zero, but friction slowly drains the x reservoir to zero.
Chapter C3  Interactions Transfer Momentum

(It may help you to remember that we have to treat the square root of a sum or difference in the same way, since \( \sqrt{a \pm b} \neq \sqrt{a} \pm \sqrt{b} \), so treat vector magnitudes as you would treat square roots.)

5. Components are not vectors. A vector \( \vec{b} \)'s components \( \vec{b}_x, \vec{b}_y, \) and \( \vec{b}_z \) are scalars (simple signed numbers), not vectors. The quantities \( \vec{b}_x, \vec{b}_y, \) and \( \vec{b}_z \), on the other hand, are component vectors, which have magnitude and direction. Keep these distinctions clear!

Observing the following general rule (which I call the column vector rule) will help you avoid making many of the specific errors listed above: If any symbol in an equation has an arrow or hat over it, you should be able to replace that symbol with a three-component column vector without changing the equation's meaning. For example, the equation \( \vec{v} = 20 \text{ m/s} \) is wrong according to this rule because a column vector is obviously not the same as a single number. The equation \( \vec{d} = \vec{d} \) is absurd according to this rule because division by a column vector makes no sense. The converse to this rule is also useful: A symbol without an arrow over it cannot be replaced by a column vector. If you are careful to observe both of these general rules when writing equations, you will sidestep the vast majority of common beginner's errors.

Exercise C3X.8

Find the mistakes (if any) in each of these statements:

a. A 2.0-kg particle moving in the \(-x\) direction at 10 m/s has momentum components \( \vec{p}_x = -20 \text{ kg m/s} \) and \( \vec{p}_y = \vec{p}_z = 0 \), so the magnitude of its momentum is \( p = -20 \text{ kg m/s} \).

b. A system consists of two particles, one moving northward with a momentum of \( \vec{p}_1 = 20 \text{ kg m/s} \) and the other westward with a momentum of \( \vec{p}_2 = 10 \text{ kg m/s} \). The system's total momentum is then \( \vec{p} = 30 \text{ kg m/s} \).

c. If a particle moves with a velocity of \( \vec{v} = 10 \text{ m/s} \) and has a momentum of \( \vec{p} = 5 \text{ kg m/s} \), then its mass must be \( m = \vec{p} / \vec{v} = 0.5 \text{ kg} \).

TWO-MINUTE PROBLEMS

C3T.1 "The car rounded the corner at a constant velocity." Would this statement make sense to a physicist?
A. No, the word velocity is being used incorrectly.
B. No, a car has to slow down to turn a corner.
C. It could make sense, or it could not, depending on the corner.
D. Yes, this statement is acceptable.

C3T.2 Imagine that a 1.0-kg cart traveling rightward at 1.0 m/s hits a 3.0-kg cart at rest. Afterward, the smaller cart is observed to move leftward with a speed of 0.75 m/s. What impulse did the collision give the smaller cart at the expense of the larger? A. None; the larger cart was at rest and so had no momentum to give.
B. The larger cart gave an impulse of the same magnitude but in the opposite direction.
C. 0.75 kg m/s leftward.
D. 1.00 kg m/s leftward.
E. 1.75 kg m/s leftward.

C3T.3 Imagine that a moving cart (cart A) hits an identical cart (cart B) at rest. Cart B remains at rest, and cart A rebounds with a speed equal to its original speed. Cart B must have participated in some other interaction during the collision process, true (T) or false (F)?

C3T.4 Imagine that two identical carts traveling toward each other at the same speed collide and come to rest. According to the momentum-transfer principle, if one of the carts is observed to be at rest after the collision, the other
A. Must be at rest also.
B. Must rebound backward with its original speed.
C. Must rebound backward with twice its original speed.
D. Must continue forward with twice its original speed.
E. Does none of the above. This process violates the momentum-transfer principle!
F. Other (specify).

C3T.5 An 8.0-kg bowling ball hits a 1.2-kg bowling pin. The force that the contact interaction exerts on the pin has the same magnitude that it exerts on the ball, T or F?

HOMEWORK PROBLEMS

Basic Skills

C3B.1 An object is observed in a certain reference frame to move from the position [-2.2 m, -3.5 m, 1.6 m] to the position [-1.8 m, 1.3 m, -0.1 m] during a time interval of 0.65 s. What are the components of the object’s velocity during this interval (assuming that the interval is sufficiently short that the velocity is essentially constant)?

C3B.2 During a certain interval of time, an object whose velocity components in a certain frame are essentially constant [1.5 m/s, -2.0 m/s, 0 m/s] moves a distance of 5.0 m. How long was the interval of time?

C3B.3 A 2.0-kg object’s momentum at a certain time is 10 kg·m/s, 37° vertically upward from due west. What are the components and magnitude of its velocity vector at this time (in a frame in standard orientation)?

C3B.4 A 1.0-kg cart traveling at 1.0 m/s rightward hits a 4.0-kg cart at rest. After the collision, the lighter cart is observed to move to the left at 0.5 m/s. What impulse did the interaction deliver to the massive cart (magnitude and direction)? What is that cart’s velocity after the collision (magnitude and direction)? Construct an arrow diagram something like the one shown in figure C3.5c.

C3B.5 Imagine that a 1.0-kg cart traveling at 1.0 m/s rightward collides with a cart at rest. If the collision interaction gives an impulse of 1.5 kg·m/s of rightward momentum to the cart originally at rest, what is the velocity of the originally moving cart after the collision?

C3B.6 The value of \( g = \text{mag}(\vec{g}) \) near the surface of the moon is about one-sixth of its magnitude near the surface of the earth. What would your mass be on the moon? What would your weight be on the moon?

C3B.7 A rocket at time \( t = 0 \) is moving at a constant velocity in deep space in the +x direction. The rocket turns 90° so that it points in the +y direction, and at \( t = 3 \) s it fires its rocket engine for 3 s. It then turns 180° to face the -y direction and at \( t = 9 \) s fires its engine again at the same thrust for 6 s.
   (a) Use either the multipart or the three-reservoir model to draw a qualitatively accurate trajectory for the rocket out to at least \( t = 18 \) s.
   (b) Is its final direction of motion the same as its initial direction?
   (c) Is the rocket pointing at the end in its final direction of motion? (Explain your results for all parts.)

C3B.8 Each of the equations or statements listed below is incorrect in some way. Describe the error in each case.
   (a) Object A has a momentum of \( \vec{p} = 2 \) kg·m/s.
   (b) At the time in question, the object’s s velocity was \( \vec{v} = -10 \) m/s.

C3T.6 It is possible for a human being to have a weight of 150 kg, T or F?

C3T.7 A cup sitting on a table constantly receives upward momentum from the table, T or F?

C3T.8 A particle is launched horizontally with an initial speed of 5 m/s and subsequently interacts only gravitationally with the earth. According to the three-reservoir model, the horizontal component of the particle’s velocity after a few seconds has passed is
   A. Somewhat greater than 5 m/s.
   B. Essentially equal to 5 m/s.
   C. Somewhat less than 5 m/s.
   D. 0.
   E. Other (specify).

C3T.9 Which of the following statements involving vectors are correct? Answer T if it is correct and F if it is not (be prepared to identify the error if you answer F).
   a. \( \vec{p}_i = \vec{p}_f + \vec{p}_t \) implies that \( \vec{p}_t = \vec{p}_i - \vec{p}_f \).
   b. \( \vec{v} = \vec{v}_i + \vec{v}_f \) implies that \( \vec{v}_i = \vec{v}_f - \vec{v} \).
   c. If \( \vec{v}_i = [0, -5.0 \text{ m/s}, 0] \), then \( \vec{v}_f = -5.0 \text{ m/s} \).
   d. If \( \vec{v}_i = [0, -5.0 \text{ m/s}, 0] \), then \( \vec{v}_t = 5.0 \text{ m/s} \).
   e. If \( \vec{v}_i = 5.0 \text{ m/s} \) and \( m = 2.0 \text{ kg} \), then \( \vec{v}_f = 10 \text{ kg·m/s} \).
(c) At the time, the object’s velocity was 25 m/s.  
(d) The displacement between points A and B is [3, 5, -1].  
(e) Particle A’s momentum is \( p_x = 2.0 \text{ kg m/s} \) west while particle B’s is \( p_y = 2.0 \text{ kg m/s} \) eastward. The magnitude of the sum of these moments is 4.0 kg m/s.

**Synthetic**

**C3S.1** At a certain time, a car is 150 m due west of your house. If it is traveling with a constant velocity of 30 m/s 30° south of east, what are the magnitude and the direction of its position relative to your house 5 s later?

**C3S.2** A cart with a mass \( m \) moving to the right with an initial speed \( v_0 \) hits a cart with mass \( M \) at rest. After the collision, the first cart rebounds to the left with a speed of \( v_x = \frac{1}{2} v_0 \), and the other cart moves to the right with a speed of \( v_y = \frac{1}{2} v_0 \). Construct a momentum-arrow diagram analogous to figure C3.5c, and use this to infer the value of the unknown mass \( M \) (express it as some multiple of \( m \)).

**C3S.3** A cart with mass \( m \) moving to the right at an initial speed of \( v_0 \) hits a cart with mass 2\( m \) that is moving at a speed of \( \frac{3}{2} v_0 \) to the left. Imagine that the carts stick together after the collision. Construct a momentum-arrow diagram analogous to figure C3.5c, and use this to infer the carts’ common final velocity \( v_f \) (magnitude and direction).

**C3S.4** A book sitting at rest on a table receives a downward impulse of 18.6 kg m/s every second from its gravitational interaction with the earth.  
(a) What is the magnitude of the gravitational force acting on the book (i.e., the book’s weight)?  
(b) What is the book’s mass (assuming that it is near the earth’s surface)?  
(c) What impulse (magnitude and direction) does the book’s contact interaction with the table exert on the book every second?

**C3S.5** Answer the questions posed in the Aristotelian Thinking Diagnostic Test at the end of chapter C2, carefully explaining your answers by using some kind of momentum flow model. (In principle, you can use such a model to correctly answer all of them, although some are still tricky.)

**C3S.6** A tugboat of mass \( m = 20,000 \text{ kg} \) pushes a barge with a mass of 80,000 kg so that starting from rest, they reach a speed of 2.0 m/s after 10 s. Assume that the effects of gravity on the tug and barge are canceled by the effects of their supporting contact interactions with the water; and assume that friction interactions are negligible and that all forces are constant during the 10-s interval. The front bumper on the tug might buckle if a force of more than 150,000 N acts on it, and its propeller might fracture if it is asked to provide more than 250,000 N of force. Is this tug operating within these limits? Answer as follows:  
(a) Start with the barge. How much impulse did the contact interaction with the tug give the barge during the time interval in question? Argue that your answer implies that the barge’s contact interaction with the tug must have exerted a force of magnitude 160,000 N on the barge, and explain carefully why this means that the force exerted on the tug’s bumper also has a magnitude of 160,000 N, violating the limit.  
(b) Argue that the tug must therefore get a backward impulse of 1,600,000 kg m/s from the barge during this interval. Yet you can calculate the tug’s actual net change in momentum during the interval from the information given. Using an arrow diagram, find the magnitude of the impulse that it must receive from the propeller, and show that this implies a propeller force of 200,000 N, which is within the operating limits of the propeller.  
(c) This result assumes no friction. Carefully explain, using a momentum-transfer model, how including friction would qualitatively affect these results. (That is, will the forces involved get larger or smaller? How do you know?)

**Rich-Context**

**C3R.1** You are the pilot of a jet plane traveling due north at 250 m/s. At a certain time, you see another plane at the same altitude at the 4 o’clock position relative to you and about 5 km away. Thirty seconds later, this plane is at the 3 o’clock position relative to you and 5 km away. Are you in danger of colliding with this plane?

**C3R.2** Perhaps you have felt a garden hose exert a backward force on your hand when you are using a nozzle that creates a high-speed spray of water. Roughly estimate how long it would take the water coming out of a typical garden-hose nozzle to fill up a 2-quart jar (\( \approx 2 \text{ liters} \approx 2000 \text{ cm}^3 \)); also make a guess about the
water's speed in meters per second. Use your estimation of the fact that 1 cm³ of water has a mass of 1 g to estimate the magnitude of force that the contact interaction between the water and nozzle must exert on the outgoing water. Explain carefully why this is also the force that your hand must exert on the nozzle if you hold it at rest. Does your answer seem reasonable, in your experience?

ANSWERS TO EXERCISES

C3X.1 No, because the particle's direction of motion is changing with time.

C3X.2 \( \vec{v} = [16 \text{ m/s}, -4.0 \text{ m/s}, -0.4 \text{ m/s}], \vec{v} = 16.5 \text{ m/s} \)

C3X.3 The originally moving cart gains \( \frac{1}{2} m_v \) of backward momentum, and the other cart gains the same amount of forward momentum. The picture looks like this:

Advanced

C3A.1 The duration of a "sufficiently short" time interval depends on the accuracy you require. Imagine that an object travels at a constant speed once around a circle in time \( \Delta t \). How big can \( \Delta t \) be (as a fraction of \( \Delta t \)) if the magnitude of \( \vec{a} \) in equation C.3.2 is to be correct within 0.1%?

C3X.4 According to the diagram, the impulse on the collision interaction contributes to the front cart is \( \vec{I} = \frac{1}{2} m_f \) forward = 1.33 kg m/s forward. If this is constant during the collision, then \( \vec{F}_c = \vec{I} / \Delta t \), so \( \text{mag}(\vec{F}_c) = \text{mag}(\vec{I}) / \Delta t = \text{mag}(\vec{I}) \Delta t = (1.33 \text{ kg m/s})/(0.01 \text{ s}) = 2.3 \text{ kg m/s}^2 = 27 \text{ N} \).

C3X.5 The unknown mass is

\[
m_2 = m_1 \frac{v_1}{v_2} = (0.5 \text{ kg}) \frac{3.0 \text{ m/s}}{2.0 \text{ m/s}} = 0.75 \text{ kg}
\]

(C3.9)

C3X.6 My weight is

\[
155 \text{ lb} \cdot \frac{4.45 \text{ N}}{1 \text{ lb}} = 680 \text{ N}
\]

(C3.10)

The weight of an adult human being generally lies between 400 and 1400 N.

C3X.7 A multi-look picture will look something like that shown to the right. Each tap transfers a bit of eastward momentum (colored arrow) to the ball which, when added to its original momentum (gray arrow), produces the ball's new momentum after each tap. We see that the ball's path bends eastward at a steadily increasing angle. In the three-reservoir model, the ball's x reservoir is steadily filled by its contact interaction with the blowing air. The ball's z reservoir is simultaneously emptied and filled at the same rate by the ball's gravitational interaction with the earth and its contact interaction with the table; as a result, its level remains fixed at zero. The y (north) reservoir is unaffected: its level remains fixed and positive throughout.

C3X.8 (a) The momentum components are written incorrectly with arrows, and the magnitude of the momentum vector is quoted as being negative. (b) The magnitude of a sum of momentum vectors is not equal to the sum of the magnitudes. (c) Vectors are set equal to scalar values, and the final equality involves division by a vector.