• C9 – Rotational Energy
Rotational Energy

Which object will be moving faster at the bottom?
\[ \theta \equiv \frac{s}{r} \]
Angular velocity

- A particle’s rotational speed

\[ v = \frac{ds}{dt} = r \frac{|d\theta|}{dt} = r\omega \]
Angular velocity

- The direction of angular velocity is defined by convention to be the direction along the axis indicated by your right thumb if you wrap your right fingers around the axis in the direction that the object’s particles move as the object rotates.
Moment of Inertia

- Moment of Intertia

\[ I = m_1r_1^2 + m_2r_2^2 + \ldots + m_Nr_N^2 = \sum_{i=1}^{N} m_ir_i^2 \]

- Object’s Rotational Kinetic Energy

\[ K_{rot} = \frac{1}{2} I \omega^2 \]

- Review exercises
  - C9.1, 2, 3

<table>
<thead>
<tr>
<th>Object and Axis</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>hoop</td>
<td>( MR^2 )</td>
</tr>
<tr>
<td>cylinder or disk</td>
<td>( \frac{1}{2}MR^2 )</td>
</tr>
<tr>
<td>thin rod</td>
<td>( \frac{1}{12}ML^2 )</td>
</tr>
<tr>
<td>thin spherical shell</td>
<td>( \frac{2}{3}MR^2 )</td>
</tr>
<tr>
<td>solid sphere</td>
<td>( \frac{2}{5}MR^2 )</td>
</tr>
</tbody>
</table>

(All rotation axes here go through the object’s center of mass.)
Translation and Rotation

- An object’s total kinetic energy is the sum of it’s translational and rotational kinetic energies

\[ K = K_{cm} + K_{rot} = \frac{1}{2} Mv_{CM}^2 + \frac{1}{2} I\omega^2 \]
Rolling without slipping

- When an object is rolling without slipping, the center of mass speed equals its rotational speed.

\[ v_{CM} = R\omega \]

Figure C9.10
(a) If the object moves without rotating, the velocity of point A is the same as that of the center of mass. (b) If the object were to rotate without moving, the velocity of point A would be \( \vec{v}_r \), where \( \text{mag}(\vec{v}_r) = R\omega \). (c) If the object moves and rotates, the velocity of point A is \( \vec{v}_r + \vec{v}_{CM} \).
Conservation of Energy - Ex. C9.4

- A cylindrical hoop with a mass of 2.5 kg and a radius of 0.50 m rolls from rest down a hill 25 m tall. How fast is the hoop rotating (in revolutions per second when it reaches the bottom? How fast is the hoop’s center of mass moving at this point?
Translation

Initial:
\[ z_i = 0 \]
\[ v_i = 0 \]
\[ z_i = 0 \]
\[ \omega_i = 0 \]

Final:

Known:
\[ v_f = 0 \]
\[ z_i = 0 \]
\[ m = 2.5 \text{ kg} \]
\[ R = 0.5 \text{ m} \]
\[ z_f = -25 \text{ m} \]

(Rotational velocity = \( \omega_f \))

Conceptual Model Diagram

Floats in space

CoE: system floats in space

\[ 0 = \Delta K_{cm} + \Delta K_{rot} + \Delta K_e + \Delta V \]
\[ = \frac{1}{2} M(\omega_f)^2 - \frac{1}{2} Mv_f^2 + \frac{1}{2} I(\omega_f)^2 - \frac{1}{2} I(\omega_f + \omega_e)^2 + 0 + Mg(z_f - z_i) \]

(Note that \( M \) and \( R \) cancel)

\[ I = \frac{M}{R^2} \]
\[ \omega_f = \frac{v_f}{R} \]

Hoop I

Assume rolling without slipping

Earth is very massive

Hoop is always near the earth
Group Problem

• C9R.1
Group Activity - the race of the shapes

- A hoop, a disk and a sphere of equal radius are having a race down an incline. Who will win? Justify your answer using conservation of energy. Show all your work!!

- Then, race the shapes down the incline to see if your calculation matches your observation.