• N12 - Introduction to Orbits

• N13 - Planetary Motion
Kepler’s Three Laws

• The orbits of the planets are ellipses, with the sun at one focus

• The line from the sun to a planet sweeps out equal areas in equal times

• The square of a planet’s period is proportional to the cube of the semimajor axis $a$ of its orbit

\[ T^2 \propto a^3 \]
Orbits Around a Massive Primary

- Two objects interacting gravitationally
- Origin is system’s center of mass
- This is true even if the system is in an external gravitation field - such as earth/moon
- Large mass is “primary, smaller mass is a “satellite”
Implications for massive object

- The position of the massive object is \( \vec{R} \approx 0 \)
- This object is at rest at the origin: \( \vec{V} \approx 0 \)
- It’s kinetic energy is negligible \( K_M \approx 0 \)
- It’s angular momentum is negligible \( \vec{L}_M \approx 0 \)
- The objects’ separation distance \( \approx \) distance of lighter object from the origin
Kepler’s Second Law

• The object’s radius vector sweeps out equal areas in equal times
Circular Orbits and Kepler’s 3rd Law

- Law of Universal Gravitation
- Assume \( M \gg m \)

\[ F_g = \frac{GMm}{r^2} \]

- \( M \) is the mass of primary, \( m \) is mass of satellite, \( r \) is distance between objects, \( G \) is the universal gravitational constant
Kepler’s 3rd Law - con’t

• Using Newton’s 2nd law and assuming circular orbits

\[ ma = F_{net} = \frac{GMm}{R^2} \]

\[ a = \frac{v^2}{R} \]

\[ \frac{v^2}{R} = \frac{GM}{R^2} \]

\[ v = \sqrt{\frac{GM}{R}} \]

\[ T = \frac{2\pi R}{v} \]

\[ T^2 = \frac{4\pi^2}{GM} R^3 \]
Ellipses

Vertices - $P_1$ and $P_2$
Foci - $F_1$ and $F_2$
Semimajor axis - $a$
Semiminor axis - $b$

$$r_1 + r_2 = 2a$$

$$
\varepsilon = \frac{c}{a} \quad 0 \leq \varepsilon < 1
$$
Hyperbolas

Vertex - $P_1$ closest point on $P$ to center

Foci - $F_1$ and $F_2$

Center is halfway between foci

\[ r_2 - r_1 = 2a \]

\[ \varepsilon = \frac{c}{a} \]

\[ \cos \theta = \frac{a}{c} = \frac{1}{\varepsilon} \]
Newton - Trajectory Diagrams for Orbits

Initial velocity

Initial Position

Forces acting on object

Time step

Don’t forget sign on $a$

For the sun, calculate $GM$ in terms of the earth-sun distance

$$GM = 4\pi^2 \frac{R^3}{T^2} = 4\pi^2 \frac{(1AU)^3}{(1y)^2} = 4\pi^2 \frac{AU^3}{y^2} = 39.48 \frac{AU^3}{y^2}$$
Use Newton to determine the shape of the earth’s orbit

Use $r_0$ is 1 AU and $v_0 = 2\pi$ AU/y

Is it circular or an ellipse?

If you increase the velocity to 8 AU/y,

how does the shape change?

What happens when you add drag?

Do exercises N13X.6 and 8
Group Problems

- N12B.2
- N12B.3
- N12S.4
- N13S.2
- N13S.8