The Gravitational Field

- Exists at every point in space
- The gravitational force experienced by a test particle placed at that point divided by the mass of the test particle
- Points in the direction of the acceleration a particle would experience if placed in that field

\[ g = \frac{F_g}{m} = -\frac{GM}{r^2} \hat{r} \]

Gravitational Potential Energy

- The gravitational force is conservative
- The gravitational force is a central force
  - It is directed along a radial line toward the center
  - Its magnitude depends only on \( r \)
  - A central force can be represented by

\[ F(r) \hat{r} \]

Gravitational Potential Energy – Work

- A particle moves from A to B while acted on by a central force \( F \)
- The path is broken into a series of radial segments and arcs
- Because the work done along the arcs is zero, the work done is independent of the path and depends only on \( r_f \) and \( r_i \)

Gravitational Potential Energy – Work

- The work done by \( F \) along any radial segment is

\[ dW = F \cdot d\mathbf{r} = F(r)dr \]

- The work done by a force that is perpendicular to the displacement is 0
- The total work is

\[ W = \int_{r_i}^{r_f} F(r) dr \]

- Therefore, the work is independent of the path
Gravitational Potential Energy

- As a particle moves from A to B, its gravitational potential energy changes by

\[ \Delta U = U_B - U_A = -\int_{r_i}^{r_f} F(r) \, dr \]

- This is the general form, we need to look at gravitational force specifically

Grav. Potential Energy for the Earth

- Choose the zero for the gravitational potential energy where the force is zero
  - This means \( U_i = 0 \) where \( r_i = \infty \)

\[ U(r) = -\frac{GM_em}{r} \]

- This is valid only for \( r \geq R_E \) and not valid for \( r < R_E \)
- \( U \) is negative because of the choice of \( U_i \)

Graph of the gravitational potential energy \( U \) versus \( r \) for an object above the Earth’s surface

- The potential energy goes to zero as \( r \) approaches infinity
- What about inside Earth?
  - only mass interior is important

Gravitational Potential Energy

- For any two particles, the gravitational potential energy function becomes

\[ U = -\frac{Gm_1m_2}{r} \]

- The potential energy is negative because the force is attractive and we chose the potential energy to be zero at infinite separation
- An external agent must do positive work to increase the separation between two objects
Binding Energy

- The absolute value of the potential energy can be thought of as the binding energy. If an external agent applies a force larger than the binding energy, the excess energy will be in the form of kinetic energy of the particles when they are at infinite separation.

Systems with Three or More Particles

- The total gravitational potential energy of the system is the sum over all pairs of particles.
- Gravitational potential energy obeys the superposition principle.
- Assuming three particles:
  \[ U_{\text{total}} = U_{12} + U_{13} + U_{23} \]
  \[ = -G \left( \frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}} \right) \]
- The absolute value of \( U_{\text{total}} \) represents the work needed to separate the particles by an infinite distance.

Energy and Satellite Motion

- Total energy \( E = K + U \)

  \[ E = \frac{1}{2} m v^2 - G \frac{Mm}{r} \]

- The absolute value of \( E \) is equal to the binding energy of the system.
  - \( E < 0 \): a bound system
  - \( E = 0 \): barely unbound
  - \( E > 0 \): unbound

Energy in a Circular Orbit

- An object of mass \( m \) is moving in a circular orbit about \( M \).
- The gravitational force supplies a centripetal force.

  \[ E = -\frac{GMm}{2r} \]
**Escape Speed from Earth**

- Use energy considerations to find the minimum value of the initial speed needed to allow the object to move infinitely far away from the Earth.
- This minimum speed is called the escape speed.

\[ v_{esc} = \sqrt{\frac{2GM}{R}} \]

- Note:
  - \( v_{esc} \) is independent of the mass of the object.
  - Independent of the direction of the velocity.
  - Ignores air resistance.

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**Escape Speed, General**

- The Earth’s result can be extended to any planet.

\[ \frac{1}{2}mv^2 = \frac{GMm}{r} \]

\[ v_{esc} = \sqrt{\frac{2GM}{R}} \]

- The table at right gives some escape speeds from various objects.

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**Planetary Atmospheres**

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**Black Holes**

- A black hole is the remains of a star that has collapsed under its own gravitational force.
- The escape speed is very large due to the concentration of a large mass into a sphere of very small radius.
- If the escape speed exceeds the speed of light, radiation cannot escape and it appears black.
- Schwarzschild radius, \( R_s \):
  - The critical radius at which the escape speed equals \( c \).
- Event horizon:
  - The imaginary surface of a sphere with radius, \( R_s \).
  - The limit of how close you can approach the black hole and still escape.
Black Holes and Accretion Disks

- Although light from a black hole cannot escape, light from events taking place near the black hole should be visible.
- If a binary star system has a black hole and a normal star, the material from the normal star can be pulled into the black hole.
- This material forms an accretion disk around the black hole.
- Friction among the particles in the disk transforms mechanical energy into internal energy.

Black Holes and Accretion Disks

- The orbital height of the material above the event horizon decreases and the temperature rises.
- The high-temperature material emits radiation, extending well into the x-ray region.
- These x-rays are characteristics of black holes.
- Examples:
  - X-ray binaries
  - $M_{BH} \sim$ a few times $M_{\odot}$
  - death state of very massive star

Black Holes at Centers of Galaxies

- There is evidence that supermassive black holes exist at the centers of galaxies, probably all galaxies.
- Theory predicts jets of materials should be evident along the rotational axis of the black hole.
- Examples:
  - Active Galactic Nuclei
    - $M_{BH} \sim$ a few million times $M_{\odot}$
    - found in centers of galaxies
    - originated from first stars, grew through merging
  - Quasars
    - $M_{BH} \sim$ a few billion times $M_{\odot}$
    - also found in galactic centers
    - originated from first stars, more merging?
    - outshine entire galaxy!

An HST image of the galaxy M87. The jet of material in the right frame is thought to be evidence of a supermassive black hole at the galaxy’s center.