**Uniform Circular Motion (Ch 6)**
- A force, $F_r$, is directed toward the center of the circle.
- This force is associated with an acceleration, $a_c$.
- Applying Newton’s Second Law along the radial direction gives:
  \[ \sum F = ma_c = m \frac{v^2}{r} \]

**Uniform Circular Motion, cont**
- A force causing a centripetal acceleration acts toward the center of the circle.
- It causes a change in the direction of the velocity vector.
- If the force vanishes, the object would move in a straight-line path tangent to the circle.

**Centripetal Force**
- The force causing the centripetal acceleration is sometimes called the **centripetal force**.
- This is not a new force, it is a new role for a force.
- It is a force acting in the role of a force that causes a circular motion.

**Conical Pendulum (HW Prob 9)**
- The object is in equilibrium in the vertical direction and undergoes uniform circular motion in the horizontal direction.
- Newton’s 2nd Law says:
  \[
  \begin{align*}
  \sum F_z &= 0 \\
  \sum F_x &= m \frac{v^2}{r}
  \end{align*}
  \]
Motion in a Horizontal Circle

- The speed at which the object moves depends on the mass of the object and the tension in the cord
- The centripetal force is supplied by the tension

Horizontal (Flat) Curve

- The force of static friction supplies the centripetal force
- The maximum speed at which the car can negotiate the curve is given by:

\[ \Sigma F_x = m \frac{v^2}{r} \]

\[ \Sigma F_y = 0 \]

\[ F_f = m \frac{v^2}{r} \]

\[ N - mg = 0 \]

\[ \mu_f N = m \frac{v^2}{r} \]

- Solve for \( v \). Does it depend on the mass of the car?

Banked Curve

- These are designed with friction equaling zero
- There is a component of the normal force that supplies the centripetal force

\[ \Sigma F_x = m \frac{v^2}{r} \]

\[ \Sigma F_y = (\cos \theta - \sin \theta) N \]

\[ N_x = m \frac{v^2}{r} \]

\[ N_y = N \sin \theta \]

\[ N = N \cos \theta \]

Loop-the-Loop

- This is an example of a vertical circle
- At the bottom of the loop (b), the upward force experienced by the object is greater than its weight

\[ \Sigma F_x = m \frac{v^2}{r} \]

\[ N - mg = m \frac{v^2}{r} \]
Loop-the-Loop, Part 2

- At the top of the circle (c), the force exerted on the object is less than its weight

\[ \Sigma F_y = m \frac{v^2}{r} \]
\[ N + mg = m \frac{v^2}{r} \]

Non-Uniform Circular Motion

- The acceleration and force have tangential components
- \( F_r \) produces the centripetal acceleration
- \( F_t \) produces the tangential acceleration
- \( \Sigma F = \Sigma F_r + \Sigma F_t \)

Vertical Circle with Non-Uniform Speed

- The gravitational force exerts a tangential force on the object
  - Look at the components of \( F_g \)
- The tension at any point can be found

\[ \Sigma F_t = m \frac{v^2}{r} \]
\[ T - mg \cos \theta = m \frac{v^2}{r} \]
\[ T = mg \cos \theta + m \frac{v^2}{r} \]

Top and Bottom of Circle

- The tension at the bottom is a maximum
- The tension at the top is a minimum
- If \( T_{\text{eq}} = 0 \), then

\[ \Sigma F_y = m \frac{v^2}{r} \]
\[ mg = m \frac{v^2}{r} \]

you can then solve for minimum speed at top (HW Prob 17)
Motion in Accelerated Frames

- A **fictitious force** results from an accelerated frame of reference
  - A fictitious force appears to act on an object in the same way as a real force, but you cannot identify a second object for the fictitious force

“Centrifugal” Force

- From the frame of the passenger (b), a force appears to push her toward the door
- From the frame of the Earth, the car applies a leftward force on the passenger
- The outward force is often called a **centrifugal force**
  - It is a fictitious force due to the acceleration associated with the car’s change in direction

“Coriolis Force”

- This is an apparent force caused by changing the radial position of an object in a rotating coordinate system
- The result of the rotation is the curved path of the ball

Fictitious Forces, examples

- Although fictitious forces are not real forces, they can have real effects
- Examples:
  - Objects in the car do slide
  - You feel pushed to the outside of a rotating platform
  - The Coriolis force is responsible for the rotation of weather systems and ocean currents
Fictitious Forces in Linear Systems

- The inertial observer (a) sees
  \[ \sum F_x = T \sin \theta = ma \]
  \[ \sum F_y = T \cos \theta - mg = 0 \]

- The noninertial observer (b) sees
  \[ \sum F'_x = T \sin \theta - F_{ centrifugal} = ma \]
  \[ \sum F'_y = T \cos \theta - mg > 0 \]

Fictitious Forces in a Rotating System

- According to the inertial observer (a), the tension is the centripetal force
  \[ T = \frac{m v^2}{r} \]

- The noninertial observer (b) sees
  \[ T - F_{ centrifugal} = T - \frac{m v^2}{r} = 0 \]

Motion with Resistive Forces

- **NOTE**: this will not be on exam
- Motion can be through a medium
  - Either a liquid or a gas
  - The medium exerts a resistive force, \( R \), on an object moving through the medium
  - The magnitude of \( R \) depends on the medium
  - The direction of \( R \) is opposite the direction of motion of the object relative to the medium
  - \( R \) nearly always increases with increasing speed

Air Resistance: \( R \) Proportional To \( v^2 \)

- For objects moving at high speeds through air, the resistive force is approximately equal to the square of the speed
  \[ R = \frac{1}{2} D \rho A v^2 \]
  - \( D \) is a dimensionless empirical quantity that called the drag coefficient
  - \( \rho \) is the density of air
  - \( A \) is the cross-sectional area of the object
  - \( v \) is the speed of the object
R Proportional To $v^2$, example

- Analysis of an object falling through air accounting for air resistance

\[
\sum F = mg - \frac{1}{2} \rho A v^2 = ma
\]
\[
a = g - \left( \frac{\rho A}{2m} \right) v^2
\]

Terminal Speed

- The terminal speed will occur when the acceleration goes to zero
- Solving the equation gives

\[
v_T = \sqrt{\frac{2mg}{\rho A}}
\]

Some Terminal Speeds

Table 6.1

<table>
<thead>
<tr>
<th>Object</th>
<th>Mass (kg)</th>
<th>Cross-Sectional Area (m²)</th>
<th>$v_T$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sky diver</td>
<td>75</td>
<td>0.20</td>
<td>60</td>
</tr>
<tr>
<td>Baseball (radius 5.7 cm)</td>
<td>0.145</td>
<td>$4.2 \times 10^{-3}$</td>
<td>45</td>
</tr>
<tr>
<td>Golf ball (radius 2.1 cm)</td>
<td>0.046</td>
<td>$1.4 \times 10^{-3}$</td>
<td>44</td>
</tr>
<tr>
<td>Haltione (radius 0.50 cm)</td>
<td>4.8 $\times 10^{-4}$</td>
<td>$7.0 \times 10^{-3}$</td>
<td>14</td>
</tr>
<tr>
<td>Raindrop (radius 0.20 cm)</td>
<td>5.4 $\times 10^{-3}$</td>
<td>$1.3 \times 10^{-2}$</td>
<td>9.0</td>
</tr>
</tbody>
</table>