Introduction to Waves

• A wave is a disturbance that moves through a medium while the medium remains essentially at rest

• Examples
  • Water, sound, tension, seismic
Wave Motion

Velocity of rope particle

Velocity of wave

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Wave Motion
Sinusoidal Wave

- One-dimensional waves
  \[ f(x,t) = A \sin(kx - \omega t) \]

  \[ k \equiv \frac{2\pi}{\lambda} \quad \omega \equiv \frac{2\pi}{T} \quad f \equiv \frac{\omega}{2\pi} = \frac{1}{T} \]

- Symbols
  - A amplitude, k wavenumber, \( \lambda \) wavelength, \( \omega \) angular frequency, \( T \) period, \( f \) frequency
Wave Characteristics

\[ \lambda = \text{wavelength} \]
\[ A = \text{amplitude} \]
\[ f = \text{frequency} \]
\[ T = \text{period} = \frac{1}{f} \]
\[ \omega = 2\pi f = \text{angular frequency} \]
Velocity of Waves

- For all waves:

\[ v = f \lambda = \frac{\omega}{k} = \frac{\lambda}{T} \]

- For a wave on a string or cord (string instruments):

\[ v = \sqrt{\frac{F_T}{m/L}} \]

where:
- \( F_T \) = tension in string
- \( m = \) mass of string
- \( L = \) length of string
- \( m/L \) may also be written as \( \mu \)
Wave Equation

\[ 0 = b \frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial x^2} \]

\[ v = \frac{1}{\sqrt{b}} \]
Solution to the wave equation

\[ f(x,t) = A \sin(kx - \omega t) \]

\[
\frac{df(x,t)}{dt} = -\omega A \cos(kx - \omega t) \\
\frac{d^2 f(x,t)}{dt^2} = \omega^2 A \sin(kx - \omega t)
\]

\[
\frac{df(x,t)}{dx} = kA \cos(kx - \omega t) \\
\frac{d^2 f(x,t)}{dt^2} = -k^2 A \sin(kx - \omega t)
\]

- Plugging into the wave equation,

\[
0 = b \frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial x^2}
\]

\[
0 = \omega^2 A \sin(kx - \omega t) - (-)k^2 A \sin(kx - \omega t)
\]

\[
b = \frac{\omega^2}{k^2} = v^2
\]
Graphs

- The graphs show:
  - (a) displacement as a function of time
  - (b) velocity as a function of time
  - (c) acceleration as a function of time

- velocity is 90° out of phase with the displacement

- acceleration is 180° out of phase with the displacement
Plug & Chug

- (a) AM radio signals have frequencies between 550 kHz and 1600 kHz and travel with a speed of $3.00 \times 10^8$ m/s. What are the wavelengths of these signals?
- (b) On FM, the frequencies range from 88.0 MHz to 108 MHz and travel at the same speed. What are their wavelengths?
Types of waves

- **Transverse**
  - displacement is perpendicular to velocity
  - Ex - light

- **Longitudinal**
  - displacement is parallel to velocity
  - Ex - sound
Movie and Group Problems

- Mechanical Universe movie
  - Disk 10

- E15B.1, E15B.7, E15S.1
Electromagnetic Waves

- Mechanical Universe movie
  - Disk 10