In 1686, Newton published the first quantitative description of the gravitational force between any two objects. He explained that the force of gravity is directly proportional to the product of the masses, $m_1$ and $m_2$, and inversely proportional to the square of the distance between them, $R$, 

$$\vec{F} = -G \frac{m_1 m_2}{R^2} \hat{r}$$

where $G$ is the proportionality constant, later quantified in Cavendish’s experiment. This is one example of a “field”, “non-contact”, or “action-at-a-distance” force. The force of gravity acts along the radial direction, and the negative sign indicates that it is always attractive.

It is known that charged objects may also attract each other, but they may also repel each other. In 1784, Charles Augustin de Coulomb investigated the relationship between force, charge and distance, and discovered that the electric force, $F$, between two charged objects assumes a very similar formulation. For two objects of charge $q_1$ and $q_2$, separated by a distance $R$, Coulomb’s Law states the force is:

$$\vec{F} = k \frac{q_1 q_2}{R^2} \hat{r}$$

where $k$ is the experimentally derived constant of proportionality. A practical difference between these two fundamental forces is that gravity acts between any two particles with mass, whereas the electric force only acts between two charged particles. Also, the strength of the two forces is significantly different. While it is necessary to have an extremely large object such as Earth to generate a noticeable gravitational force, the electric force can be studied on the laboratory scale.

Fundamental forces interact via fields. In the case of electrostatics, fields arise from the potential for a source charge to exert a force on a test charge, by virtue of the quantity of source charge and the distance from it. We can re-write Coulomb’s Law for point charges in terms of the force $F$ between a test charge $q_1$ and an electric field $E$ that we associate with charge $q_2$:

$$\vec{F} = q_1 \vec{E}$$

where both $F$ and $E$ are vectors. From Coulomb’s Law above, the expression for the electric field of a point charge is therefore given by

$$\vec{E} = \frac{\vec{F}}{q_1}, \quad \text{therefore:} \quad \vec{E} = \frac{k q_1 q_2}{r^2} \hat{r} = k \frac{q_2}{r^2} \hat{r}$$

or more generally, the electric field due to a point charge is: $\vec{E} = k \frac{q}{r^2} \hat{r}$.

As this is a vector quantity, the field associated with a distribution of charges is determined from the trigonometric vector sum of the fields associated with each source charge.
Part I: EM Field Program

Open the program Exploration of Physics, which can be found by opening the Finder (lower left corner of the Dock) and going to Applications on the left side of the window.

Procedure:

After you open the software, go to “e & m” on the menu at the top of the window. In the menu, choose “Electric Fields.” In this part of the program, you have a graph and numerous charges that you can drag onto the graph. There are also some charge configurations available.

A. Electric Field from a single charge

1. Choose the first preset charge configuration (a single positive charge, shown as red). This will automatically place the charge at the center of the graph. You’ll notice that as soon as you placed the charge on the graph, lines appeared at the locations of the gray dots. These lines represent the direction and relative magnitude of the electric field at those points. The thicker the line, the stronger the electric field. Notice also that there is a green arrow attached to your cursor when it’s in the graph window. This also represents the direction and relative magnitude of the electric field.

2. Sketch the electric field lines of this single positive charge below, using the program as a guide. Make sure you include arrows to show the direction of the field. In most instances, you can see how the lines will connect from dot to dot throughout the graph, but be careful, electric field lines NEVER CROSS!!

3. Describe the direction and magnitude of the electric field due to this single, positive charge.

4. The box on the left side of the screen shows the location of your mouse and the magnitude of the electric field and its components at that point. Use this feature to find the electric field due to the charge at a distance of 100 m. Verify this measurement using Coulomb’s law to calculate what the value of the field should be at that distance. Use a value of $9 \times 10^9 \text{ Nm}^2/\text{C}^2$ for the Coulomb constant. Is the program calculating the field correctly?

   Value from program: $E =$ ____________ N/C
   Value from Coulomb’s law: $E =$ ____________ N/C
5. Again using the preset configurations, replace the positive charge with a single negative charge (represented by a blue dot). Sketch the electric field of this charge.

6. What is different about the electric field of the negative charge?

**B. Electric Field from a dipole**

7. Now choose the 4\textsuperscript{th} preset charge configuration, a positive charge and a negative charge placed horizontally apart from each other on the x-axis. This is known as an electric dipole. Sketch the electric field of this charge configuration.

8. Use the program to find the value of the electric field (magnitude \textit{and} direction) at the point (x,y) = (0 m, 100 m).

\[ E = \text{__________ N/C} \]

9. Use Coulomb’s law to calculate the value of the field at this point. Don’t forget that the electric field is a vector quantity! Is the program calculating the field correctly?

\[ E = \text{__________ N/C} \]
C. Electric Field from two like charges

10. For this part of the lab, manually place two positive charges on the x-axis. Place one of them at x = -100 m and the other at x = +100 m. Sketch the electric field of this charge configuration. Remember: electric field lines NEVER CROSS!

11. Without using the program, just referring to your sketch above, predict the value of the electric field at the origin.

\[ E_{\text{origin}} = \text{__________ N/C} \]

12. Verify your prediction in number 11 by using the program. Was your prediction correct?

D. Electric Force Game!!!

Go back to the E & M menu in the program and choose “Coulomb forces”. On this screen, you have a positive test charge already on the graph. The blue arrow represents the initial velocity of the charge. You can change the location of the charge and the initial velocity by dragging the charge and dragging the blue arrow.

On the right side of the window you have a menu of “Paths” to choose from. Choose Path A and a green path will be displayed on the screen.

Drag the test charge (the one that is already in the plot window) to the beginning of the green path. The coordinates should be x = -140.0 m and y = 30 m.

Now it is your task to drag and drop the appropriate charges and set the initial velocity of the test charge so that it follows the green path.

13. What charges did you place on the graph and where?

14. What did you have to set the initial velocity to in order to have the test charge follow the green path? Give both the magnitude and direction of the velocity you set.
**Practice problem**
Consider the three charges shown below. The sizes of the three charges are as follows: $q_1 = +1.0 \text{ C}, \, q_2 = -2.0 \text{ C}, \, q_3 = +1.0 \text{ C}, \, a = 12 \text{ cm}, \, b = 15 \text{ cm}.$

![Diagram of three charges with labels q1, q2, q3, and distances a and b]

a) What is the x-component of the magnitude of the net force on $q_2$?

b) What is the y-component of the magnitude of the net force on $q_2$?

c) What is the magnitude of the net force on $q_2$?

d) What angle does the net force vector make relative to the horizontal? Draw the components and the net force on $q_2$ on the diagram.