Gravitational equilibrium:
The outward push of pressure balances the inward pull of gravity.

Weight of upper layers compresses lower layers.

Gravitational equilibrium:
Energy provided by fusion maintains the pressure.

Gravitational contraction:
Provided energy that heated core as Sun was forming.
Contraction stopped when fusion began.
Stellar Structure

- Assume stars are in equilibrium
- Stars are spherical, so physics depends only on radius

Hydrostatic Equilibrium

\[ \frac{dP}{dr} = -\frac{G M(r) \rho}{r^2} \]

where

\[ M(r) = \int_0^r \rho \, dV = 4\pi \int_0^r \rho \, r^2 \, dr \]

Need \( \rho(r) \) to determine \( M(r) \) and then \( P(r) \)

Check

- Calculate the average density of the sun.

  The central density in the sun is 110 times its average density. What is the central density?

Pressure

- Gas Thermal Pressure

\[ P = \frac{n \, k \, T}{\mu \, m_H} \]

where \( \mu = \frac{m}{m_H} = \text{ave mass particle} \)

and \( \rho = n_{\text{col}} \mu \, m_H \)
Mass Fractions

- We can express mean molecular weight in terms of mass ratios, or **mass fractions**:

\[
X \equiv \frac{\text{total mass of hydrogen}}{\text{total mass of gas}} \\
Y \equiv \frac{\text{total mass of helium}}{\text{total mass of gas}} \\
Z \equiv \frac{\text{total mass of metals}}{\text{total mass of gas}}
\]

Mean Molecular Weight

- For a neutral gas:

\[
\frac{1}{\mu} \approx X + \frac{1}{4} Y + < \frac{1}{A} >_n Z
\]

- For an ionized gas:

\[
\frac{1}{\mu_i} \approx 2X + \frac{3}{4} Y + < \frac{1+z}{A} >_i Z
\]

where \( A = \# \) of protons + \# of neutrons

Pressure

- Radiation Pressure
  - photons have momentum

\[
P_{rad} = \frac{4 \sigma T^4}{3c}
\]

Central pressure in Sun

- The pressure in the center of the sun equals the weight per unit area of the material on top. We can approximate central pressure:

\[
P_c = \frac{M_{sun}}{4\pi R_{sun}^2} g \sim (M_{sun}/R_{sun}^2)(G M_{sun}/R_{sun}^3)
\]

- A more accurate calculation yields:

\[
P_c = 19 (G M_{sun}^3/R_{sun}^4) = ?
\]
Central Temperature in Sun

- Group problem:
  - Using our estimates for central pressure and density, what is central temperature of the sun? Assume the sun consists of 90% H, 10% He (ignore the 1% heavier elements), and that everything is ionized in the core.
  - What is peak wavelength of blackbody spectrum for this temperature?

Radiative Transfer

- You should find the radiation field in core contains mostly x-rays
- on average, photons travel 0.5 cm before scattering
- random walk out to surface of sun
  - average energy drops from x-ray to visible
  - at photosphere, photons go from walking to flying
  - if luminosity transfer is too high for radiative transfer, convection results

Think-Pair-Share

- The total photon flight time is
  \[ t = N \frac{1}{v} = 3 \frac{R_s^2}{v} \]
  Compute the flight time assuming \(l=0.5\) cm and compare this with a free-flight time, \(R/c\), if photons could fly straight from the center of the sun to the surface.

Source of Sun’s Energy

- In the sun, the thermal energy of a plasma is \(10^3\) times greater than instantaneous energy content in radiation field.
- Photons leak out in \(3 \times 10^4\) yr.
- Current thermal energy would leak away in \(10^9 \times 3 \times 10^4\) yr = 3 \(\times 10^7\) yr.
- Must be another energy source!
Gravitational Energy?
- Kelvin-Helmholtz, nineteenth century
- Virial theorem:
  - thermal E + grav. pot. E = E = 1/2 grav. pot. E
- If sun contracted from much larger to its present size, it would yield
  \[ E_{\text{thermal}} \sim \frac{1}{2} \frac{G M^2}{R} \sim 2 \times 10^{48} \text{ ergs} \]
- Assuming current energy output, how long would sun shine?

Energy Requirements
- We know the Earth is ~5 billion years old. We can estimate total energy output of sun at:
  \[ E_{\text{tot}} = L_\odot \text{ lifec} = \]
- The number of atoms in the sun is:
  \[ N_{\text{atoms}} = \frac{2 \times 10^{33} \text{ g}}{1.6 \times 10^{-24} \text{ g}} \approx 10^{57} \text{ atoms} \]

Energy Requirements
- Energy required per atom:
  \[ \frac{E_{\text{total}}}{\text{atom}} \sim 10^{-6} \text{ ergs} \approx 10^6 \text{ eV} \]
- In comparison, rest-mass energy is:
  \[ \frac{E_{\text{rest-mass}}}{\text{atom}} \sim 10^9 \text{ eV} \]
- You need 0.001 m c²!

Thermonuclear Fusion
- Process by which heavy elements are built up from light ones via nuclear reactions at high temperatures.
- At center of sun, T~15 million degrees
  - only H can fuse
  - implies core must be most H
  - clue that predominant form of matter in universe is H
- Limited energy source implies finite lifetime
Controlled Fusion

- Balance between pressure and gravity
  - ideal gas
  - not true for degenerate matter
- Radius of sun maintains $T_c \rightarrow$ nuclear reaction rate to balance energy loss due to random walk leakage

How does nuclear fusion occur in the Sun?

Fission

Big nucleus splits into smaller pieces
(Nuclear power plants)

Fusion

Small nuclei stick together to make a bigger one
(Sun, stars)

High temperature enabled nuclear fusion to happen in the core

At low speeds, electromagnetic repulsion prevents the collision of nuclei.

At high speeds, nuclei come close enough for the strong force to bind them together.

Sun releases energy by fusing four hydrogen nuclei into one helium nucleus.
Proton-Proton Chain

\[ ^1_1H + ^1_1H \rightarrow ^2_2H + e^+ + \nu_e + 1.44 \text{ MeV} \quad (10^{10} \text{ yr}) \]

\[ ^2_2H + ^1_1H \rightarrow ^3_2He + \gamma + 5.49 \text{ MeV} \quad (6 \text{ sec}) \]

\[ ^3_2He + ^3_2He \rightarrow ^6_4He + ^1_1H + ^1_1H + 12.85 \text{ MeV} \quad (10^6 \text{ yr}) \]

where \( \frac{A}{2} \), \( A = \text{mass number} \), \( Z = \text{number of protons} \)

Solar Thermostat

Temperature Decreases

Fusion Rate Decreases

Temperature Restored

Core compresses

CNO Cycle

\[ ^{12}_{6}C + ^1_1H \rightarrow ^{13}_{7}N + \gamma + 1.95 \text{ MeV} \]

\[ ^{13}_{7}N \rightarrow ^{13}_{6}C + e^+ + \nu_e + 2.25 \text{ MeV} \]

\[ ^{13}_{6}C + ^1_1H \rightarrow ^{14}_{7}N + \gamma + 7.54 \text{ MeV} \]

\[ ^{14}_{7}N + ^1_1H \rightarrow ^{15}_{8}O + \gamma + 7.35 \text{ MeV} \]

\[ ^{15}_{8}O \rightarrow ^{15}_{7}N + e^+ + \nu_e + 2.71 \text{ MeV} \]

\[ ^{15}_{7}N + ^1_1H \rightarrow ^{16}_{8}O + ^1_1H + 4.95 \text{ MeV} \]

• Requires star to have some C
  - still fuses H -> He but uses C as a catalyst
  - Proposed by Hans Bethe in 1938
The Sun

Triple Alpha Reaction

\[ ^4\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma - 0.095 \text{ MeV} \]
\[ ^8\text{Be} + ^3\text{He} \rightarrow ^{12}\text{C} + \gamma + 7.4 \text{ MeV} \]

\[ \epsilon_{\alpha \alpha} \cong \frac{0.33}{\rho^2 Y^2 \rho_{\text{H}} T^{3.6}} \]

- 3-body interaction b/c Be rapidly decays if not struck with another He nuclei
- very strong T dependence
- \( T > 10^8 \text{ K} \)

Energy generation rate

More Fusion by He capture

\[ ^{12}\text{C} + ^3\text{He} \rightarrow ^{15}\text{O} + \gamma \]
\[ ^{16}\text{O} + ^3\text{He} \rightarrow ^{20}\text{Ne} + \gamma \]

- Carbon burning: \( T > 6 \times 10^8 \text{ K} \)
- Oxygen burning: \( T > 10^9 \text{ K} \)
- Only H and He made in big bang
- all other elements made in stellar interiors

Nuclear Fusion

- H fusion at 15 million K
- difficult to get from He to C
- very sensitive to T
- Heavier nuclei require higher T