3. Abstract Axiom Systems

Simply put, an axiom system consists of

- A set of undefined terms.
- A set of axioms or postulates – statements that are assumed to be true.
- An agreed upon system of logical deduction.

Propositions that can be derived from these ingredients form the body of knowledge that is encapsulated by the system.

The Elements were the first known example of such a system.

Properties

- A set of axioms is said to be *consistent* if it is not possible to deduce from the axioms a result that contradicts any axiom of previously deduced result.

- An axiom are said to be *independent* if it cannot be logically deduced from the other axioms.

- An axiom system is *complete* if it is impossible to add a new independent axiom that is consistent with the others and does not contain new undefined terms. In other words, any statement formed from the concepts of the system must be either provable or disprovable.

Euclid’s system of axioms was found to be incomplete, since it was not possible to prove or disprove the statements such as “a line joining a point inside a circle to a point outside must intersect the circle”. Within the framework that Euclid specified.

Part of the problem with the Elements was that it was assumed (since it could be empirically demonstrated) that statement such as those above were self-evident.
Many fixes to this problem have been proposed. We will use a system developed in the late 1960’s by a group of mathematicians and mathematics educators, collectively known as the School Mathematics Study Group (SMSG).

Refer to the handout.

This axiom system is complete and consistent, but it is not an independent set of axioms.