Name: ____________________________

**INSTRUCTIONS**

1. This examination is closed book and closed notes. All your belongings except a pen or pencil and a calculator should be put away and your bookbag should be placed on the floor.

2. You will find one page of useful formulae on the last page of the exam.

3. Please show all your work in the space provided on each page. If you need more space, feel free to use the back side of each page.

4. **Academic dishonesty** (i.e., copying or cheating in any way) will result in a zero for the exam, and may cause you to fail the class.

**IN ORDER TO RECEIVE MAXIMUM CREDIT,**

**EACH SOLUTION SHOULD HAVE:**

1. A labeled picture or diagram, if appropriate.
2. A list of given variables.
3. A list of the unknown quantities (i.e., what you are being asked to find).
4. One or more free-body or force-interaction diagrams, as appropriate, with labeled 1D or 2D coordinate axes.
5. Algebraic expression for the net force along each dimension, as appropriate.
6. Algebraic expression for the conservation of energy or momentum equations, as appropriate.
7. An algebraic solution of the unknown variables in terms of the known variables.
8. A final numerical solution, including units, with a box around it.
9. An answer to additional questions posed in the problem, if any.
1. A 10 m long glider with a mass of 680 kg (including the passengers) is gliding horizontally through the air at 30 m/s when a 60 kg skydiver drops out by releasing his grip on the glider. Using the appropriate conservation law, determine the glider’s velocity just after the skydiver lets go.

**Solution:**

This is a conservation of momentum problem. At the start, both the glider and the skydiver are traveling at the same speed.

\[
p_{fx} = p_{ix}
\]

\[
m_g(v_f)_g + m_s(v_f)_s = (m_g + m_s)(v_i)
\]

Immediately after release, the skydiver’s horizontal velocity is still 30 m/s. Solving for the final velocity of the glider,

\[
(v_f)_g = \frac{(m_g + m_s)(v_i) - m_s(v_f)_s}{m_g}
\]

\[
(v_f)_g = \frac{(680\text{kg})(30\text{m/s}) - 60\text{kg}(30\text{m/s})}{620\text{kg}}
\]

\[
(v_f)_g = 30\text{m/s}
\]

The skydiver’s motion in the vertical direction has no influence on the glider’s horizontal motion.
2. A 20 g ball of clay traveling east at 2.0 m/s collides with a 30 g ball of clay traveling 30° south of west at 1.0 m/s. What are the speed and the direction of the resulting 50 g ball of clay?

Solution:

This is an inelastic two-dimensional conservation of momentum problem. Applying conservation of momentum in the \(x\)-direction, the final momentum in the \(x\)-direction is

\[
p_{fx} = p_{ix} = m_1(v_{ix})_1 + m_2(v_{ix})_2
\]

\[
= (0.020 \text{ kg})(2.0 \text{ m/s}) - (0.030 \text{ kg})(1.0 \text{ m/s}) \cos 30°
\]

\[
= 0.0140 \text{ kg m/s}
\]

Applying conservation of momentum in the \(y\)-direction, the final momentum in the \(y\)-direction is

\[
p_{fy} = p_{iy} = m_1(v_{iy})_1 + m_2(v_{iy})_2
\]

\[
= (0.020 \text{ kg})(0 \text{ m/s}) - (0.030 \text{ kg})(1.0 \text{ m/s}) \sin 30°
\]

\[
= -0.0150 \text{ kg m/s}
\]

Next, we find the magnitude of the final momentum by combining the \(x\)- and \(y\)-components using the Pythagorean theorem:

\[
p_f = \sqrt{p_{fx} + p_{fy}}
\]

\[
= \sqrt{(0.0140 \text{ kg m/s})^2 + (-0.0150 \text{ kg m/s})^2}
\]

\[
= 0.0205 \text{ kg m/s}
\]
Using the final momentum, we can solve for the final speed:

\[ p_f = (m_1 + m_2)v_f \] (10)

\[ \Rightarrow v_f = \frac{p_f}{m_1 + m_2} \] (11)

\[ = \frac{0.0205 \text{ kg m/s}}{0.020 \text{ kg} + 0.030 \text{ kg}} \] (12)

\[ = 0.41 \text{ m/s} \] (13)

The direction is given by

\[ \theta = \tan^{-1}\left(\frac{p_{fy}}{p_{fx}}\right) \] (15)

\[ = \tan^{-1}\left(\frac{-0.0150 \text{ kg m/s}}{0.0140 \text{ kg m/s}}\right) \] (16)

\[ = -47^\circ \] (17)
3. Most geologists believe that the dinosaurs became extinct 65 million years ago when a large comet or asteroid struck the earth, throwing up so much dust that the sun was blocked out for a period of many months. Suppose an asteroid with a diameter of 2.0 km and a mass of $1.0 \times 10^{13}$ kg hits the earth with an impact speed of $4.0 \times 10^4$ m/s.

(a) What is the earth’s recoil speed after such a collision? *(Hint: Use a reference frame in which the earth is initially at rest.)*

(b) What percentage is this of the earth’s speed around the sun?

**Solution:**

![Pictorial representation]({"image_url"})

(a) This is an inelastic collision, and therefore a conservation of momentum problem. At the start, the asteroid has a speed of $4.0 \times 10^4$ m/s while the earth is not moving.

\[
p_{ix} = p_{fx} \tag{1}
\]

\[
M_{earth}(v_i)_e + m_a(v_i)_a = (M_{earth} + m_a)(v_f) \tag{2}
\]

Solving for the final velocity of the earth and asteroid system,

\[
v_f = \frac{M_{earth}(v_i)_e + m_a(v_i)_a}{(M_{earth} + m_a)} \tag{4}
\]
\[
\begin{align*}
\text{v}_f &= 0 \text{ kg m/s} + (1.0 \times 10^{13} \text{ kg})(4.0 \times 10^4 \text{ m/s}) \\
&\quad - (5.98 \times 10^{24} \text{ kg} + 1.0 \times 10^{13} \text{ kg}) \\
&= 6.7 \times 10^{-8} \text{ m/s} 
\end{align*}
\]

(b) The speed of the earth going around the sun is
\[
\begin{align*}
\text{v}_E &= \frac{2\pi r_{\text{earth-sun}}}{T} = \frac{2\pi (1.50 \times 10^{11} \text{ m})}{3.15 \times 10^7 \text{ s}} = 3.0 \times 10^4 \text{ m/s} 
\end{align*}
\]

Therefore,
\[
\begin{align*}
\frac{v_f}{v_E} &= (6.7 \times 10^{-8} \text{ m/s})/(3.0 \times 10^4 \text{ m/s}) \\
&= 2.2 \times 10^{-12} \\
&= 2.2 \times 10^{-10} \%. 
\end{align*}
\]
4. Fred (mass 60 kg) is running with the football at a speed of 6.0 m/s when he is met head-on by Brutus (mass 120 kg), who is moving at 4.0 m/s. Brutus grabs Fred in a tight grip, and they fall to the ground. Which way do they slide, and how far? The coefficient of kinetic friction between football uniforms and Astroturf is 0.30.

Solution:

This is an inelastic collision in which Fred and Brutus are moving toward each other. Assume that Brutus is moving in the positive \( x \)-direction and Fred is moving in the negative \( x \)-direction. Because momentum is conserved, we have

\[
\begin{align*}
I_{ix} &= I_{fx} \\
m_B(v_i)_B - m_F(v_i)_F &= (m_B + m_F)(v_f)
\end{align*}
\]  

(1)

(2)

Solving for the final velocity of Brutus and Fred,

\[
v_f = \frac{m_B(v_i)_B - m_F(v_i)_F}{m_B + m_F}
\]

(3)

\[
= \frac{(120 \text{ kg})(4.0 \text{ m/s}) - (60 \text{ kg})(6.0 \text{ m/s})}{(120 \text{ kg} + 60 \text{ kg})}
\]

(4)

\[
= 0.667 \text{ m/s}
\]

(5)

Now that we know that Brutus and Fred moved to the right after the collision, we can find the distance that they traveled. Friction slows them down, and therefore the force of friction must act in the negative \( x \)-direction. The net force is equal to the total mass (the mass of both players) times the acceleration.

\[
\sum F_x = -f_k = (m_B + m_F)a
\]

(6)

\[
-\mu n = (m_B + m_F)a
\]

(7)

\[
-\mu(m_B + m_F)g = (m_B + m_F)a
\]

(8)

\[
\Rightarrow a = -\mu g
\]

(9)

\[
= -(0.30) \times (9.8 \text{ m/s}^2)
\]

(10)

\[
= -2.94 \text{ m/s}^2
\]

(11)
Finally, we can use the kinematic equation to find the distance traveled with $v_f = 0$:

$$v_f^2 = v_i^2 + 2ad \quad (12)$$

$$\Rightarrow d = -\frac{v_i^2}{2a} \quad (13)$$

$$= -\frac{(0.667 \text{ m/s}^2)^2}{2 \times (-2.94 \text{ m/s}^2)} \quad (14)$$

$$= 0.076 \text{ m} \quad (15)$$

$$= 7.6 \text{ cm} \quad (16)$$
5. A two-stage rocket is traveling at 1200 m/s with respect to the earth when the first stage runs out of fuel. Explosive bolts release the first stage and push it backward with a speed of 35 m/s relative to the second stage. The first stage is three times as massive as the second stage. What is the speed of the second stage after the separation?

Solution:

The rocket is traveling at an initial speed before the explosion. The two parts separate after the explosion. To find the speed of the second stage, we’ll use the conservation of momentum equation.

\[
\begin{align*}
\mathbf{p}_f &= \mathbf{p}_i \quad (1) \\
(m_1 + m_2)v_i &= m_1v_{1f} + m_2v_{2f} \quad (2) \\
(3m_2 + m_2)v_i &= 3m_2v_{1f} + m_2v_{2f} = m_2(3v_{1f} + v_{2f}) \quad (3) \\
4m_2v_i &= m_2(3v_{1f} + v_{2f}) \quad (4) \\
4v_i &= 3v_{1f} + v_{2f} \quad (5)
\end{align*}
\]

The fact that the first stage is pushed backward at 35 m/s relative to the second can be written

\[
\begin{align*}
v_{1f} &= -35 \text{ m/s} + v_{2f} \quad (6) \\
4v_i &= 3(-35 \text{ m/s} + v_{2f}) + v_{2f} \quad (7) \\
4v_i &= -105 \text{ m/s} + 4v_{2f} \quad (8)
\end{align*}
\]

Solving for the velocity of the second stage \(v_{2f}\)

\[
v_{2f} = \frac{4v_i + 105 \text{ m/s}}{4} \quad (9)
\]
\[
\begin{align*}
\text{4} & = \frac{4 \times 1200 \text{ m/s} + 105 \text{ m/s}}{4} \quad \text{(10)} \\
& = 1.2 \text{ km/s} \\
\end{align*}
\]
6. A 500 g rubber ball is dropped from a height of 10 m and undergoes a perfectly elastic collision with the earth.

(a) For an elastic collision, what quantities are conserved?

(b) Write out the conservation of momentum equation for this problem.

(c) Write out the conservation of energy equation for this problem.

(d) Which quantities are not known in these equations?

(e) Explain how to obtain the formulas for these unknown quantities.

(f) What is the earth’s velocity after the collision? Assume the earth was at rest just before the collision.

(g) How many years would it take the earth to move 1.0 mm at this speed?

Solution:

(a) In an elastic collision, both energy and momentum are conserved.

(b) The conservation of momentum equation is

\[ p_i = p_f \]  \hspace{1cm} (1)

\[ m_b v_{bi} + m_e v_{ei} = m_b v_{bf} + m_e v_{ef} \]  \hspace{1cm} (2)

\[ m_b v_{bi} + 0 = m_b v_{bf} + m_e v_{ef} \]  \hspace{1cm} (3)

(c) The conservation of energy equation is

\[ K_i + U_i = K_f + U_f \]  \hspace{1cm} (4)

\[ \frac{1}{2} m_b (v_{bi})^2 + \frac{1}{2} m_e (v_{ei})^2 = \frac{1}{2} m_b (v_{bf})^2 + \frac{1}{2} m_e (v_{ef})^2 \]  \hspace{1cm} (5)

\[ \frac{1}{2} m_b (v_{bi})^2 + 0 = \frac{1}{2} m_b (v_{bf})^2 + \frac{1}{2} m_e (v_{ef})^2 \]  \hspace{1cm} (6)

\[ m_b (v_{bi})^2 + 0 = m_b (v_{bf})^2 + m_e (v_{ef})^2 \]  \hspace{1cm} (7)

(d) The quantities that are not known in these equations are \( v_{bf} \) and \( v_{ef} \).

(e) We’re given the mass of the rubber ball and it’s initial velocity. We also know the mass of the earth. Therefore, we have two unknowns and two equations. The unknowns are \( v_{bf} \) and \( v_{ef} \). To find the final velocity of the earth, we can
solve for \(v_{bf}\) in the momentum equation and plug this into the energy equation to eliminate \(v_{bf}\). This will give us the equation for \(v_{ef}\). Once we have an expression for \(v_{ef}\), we can plug this into the momentum equation to get an expression for \(v_{bf}\).

(f) We’ll assume that the earth is at rest right before the collision. To find the speed of the rubber ball right before the collision, we can use kinematics.

\[
(v_{fB})^2 = (v_{iB})^2 + 2a\Delta h = 2(9.8 \text{m/s}^2)(10 \text{m})
\]

\[
v_{fB} = \sqrt{2(9.8 \text{m/s}^2)(10 \text{m})} = 14.0 \text{m/s}
\]

By continuing with the derivations above, we find that the equations for \(v_{bf}\)are

\[
v_{bf} = \frac{m_b v_{bi} - m_e v_{ef}}{m_b}
\]

(10)

\[
v_{ef} = \frac{2m_b v_{bi}}{m_e + m_b}
\]

(11)

We found that the velocity of the ball right before it collided with the earth was 14.0 m/s. This is our initial velocity of the ball in our conservation of momentum and energy equations. So, \(v_{bi} = 14.0 \text{m/s}\). Plugging this into the above equation, we solve for \(v_{ef}\).

\[
v_{ef} = \frac{2(0.500 \text{kg})(14.0 \text{m/s})}{5.98 \times 10^{24} \text{kg}} = 2.3 \times 10^{-24} \text{m/s}
\]

(12)

(g) The time it would take the earth to move 1.0mm at this speed is given by the kinematics equation.

\[
x_f = x_i + v_i \Delta t + \frac{1}{2} a \Delta t^2 = 0 + v_i \Delta t + 0
\]

\[
\Delta t = \frac{x_f}{v_i} = \frac{0.001 \text{m}}{2.3 \times 10^{-24} \text{m/s}} = 4.3 \times 10^{20} \text{sec} = 1.36 \times 10^{13} \text{years}
\]

(13)

(14)
7. A package of mass $m$ is released from rest at a warehouse loading dock and slides down a 3.0 m high frictionless chute to a waiting truck. Unfortunately, the truck driver went on a break without having removed the previous package, of mass $2m$, from the bottom of the chute.

(a) Suppose the packages stick together. What is their common speed after the collision?

(b) For an elastic collision, what quantities are conserved?

(c) Write out the conservation of momentum equation for this problem.

(d) Write out the conservation of energy equation for this problem.

(e) Which quantities are not known in these equations?

(f) Explain how to obtain the formulas for these unknown quantities.

(g) If the collision between the packages is perfectly elastic, to what height does the package of mass $m$ rebound?

\[ \begin{align*}
K_i + U_i &= K_f + U_f \\
0 + mgh &= \frac{1}{2}mv^2 \\
v &= \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(3.0 \text{ m})} = 7.668 \text{ m/s}
\end{align*} \]

Solution:

(a) From conservation of energy, we can find the speed of the package at the bottom of the chute.

\[ \begin{align*}
K_i + U_i &= K_f + U_f \\
0 + mgh &= \frac{1}{2}mv^2 \\
v &= \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(3.0 \text{ m})} = 7.668 \text{ m/s}
\end{align*} \]

If the packages stick together, the collision is inelastic and conservation of momentum is conserved.

\[ \begin{align*}
p_i &= p_f \\
m_1v_{1i} + m_2v_{2i} &= (m_1 + m_2)v_f \\
mv_{1i} + 0 &= 3mv_f
\end{align*} \]
Cancelling the m’s
\[ v_{1i} = 3v_f \]  
(7)

Solving for \( v_f \), we find
\[ v_f = \frac{v_{1i}}{3} = \frac{7.668 \text{m/s}}{3} = 2.56 \text{m/s} \]  
(8)

The packages move together with a speed of 2.56 m/s.

(b) In an elastic collision, both energy and momentum are conserved.

(c) In this conservation of momentum equation, we’ll use the velocity calculated above for the initial velocity before the collision. Also, the package at the bottom is at rest before the collision. The conservation of momentum equation gives
\[ p_i = p_f \]  
(10)
\[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]  
(11)
\[ m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f} \]  
(12)
\[ m v_{1i} = m v_{1f} + (2m) v_{2f} \]  
(13)
\[ v_{1i} = v_{1f} + 2v_{2f} \]  
(14)
\[ v_{1f} = v_{1i} - 2v_{2f} \]  
(15)

(d) The conservation of energy equation is
\[ K_i + U_i = K_f + U_f \]  
(16)
\[ \frac{1}{2} m_1 (v_{1i})^2 + \frac{1}{2} m_2 (v_{2i})^2 = \frac{1}{2} m_1 (v_{1f})^2 + \frac{1}{2} m_2 (v_{2f})^2 \]  
(17)

(e) The quantities that are not known in these equations are \( v_{1f} \) and \( v_{2f} \).

(f) We have two unknowns and two equations. The unknowns are \( v_{1f} \) and \( v_{2f} \).
To find the final velocity of the second package, we can solve for \( v_{1f} \) in the momentum equation and plug this into the energy equation to eliminate \( v_{1f} \).
This will give us the equation for \( v_{2f} \). Once we have an expression for \( v_{2f} \), we can plug this into the momentum equation to get an expression for \( v_{1f} \).

From the derivations above, the equation relating the final speed of the first mass to it’s initial speed is
\[ v_{1f} = \frac{m - 2m}{m + 2m} v_{1i} = \frac{-1}{3} (7.668 \text{m/s}) = -2.56 \text{m/s} \]  
(19)
The package rebounds and goes back up the ramp. To determine how high the package goes, we can use the conservation of energy.

\[
\begin{align*}
K_i + U_i &= K_f + U_f \\
\frac{1}{2}mv^2 + 0 &= 0 + mgh \\
h &= \frac{v^2}{2g} = \frac{(-2.56\text{m/s})^2}{2(9.8\text{m/s}^2)} = 0.33 \text{ m}
\end{align*}
\]
8. A roller coaster car on the frictionless track shown below starts from rest at height $h$. The track’s valley and hill consist of circular-shaped segments of radius $R$.

(a) What is the maximum height $h_{\text{max}}$ from which the car can start so as not to fly off the track when going over the hill? Give your answer as a multiple of $R$. Hint First find the maximum speed for going over the hill.

(b) Evaluate $h_{\text{max}}$ for a roller coaster that has $R = 10$ m.

Solution:

(a) When the car is in the valley and on the hill it can be considered to be in circular motion. We start at the top of the hill and determine the maximum speed so that the car stays on the track. We can use the equations for circular motion. The acceleration is given by

$$ a = -\frac{v^2}{r}, \quad (1) $$

where the minus sign means that the centripetal acceleration points inward, toward the center of the circular track.

The free-body diagram for the car at the top of the hill illustrates the two forces acting on the car. Using Newton’s second law, we know

$$ \sum F = ma = -m\frac{v^2}{R} \quad (2) $$

$$ F_N - F_g = -m\frac{v^2}{R} \quad (3) $$

When the normal force is greater than zero, the car is still touching the track. Therefore, by setting the normal force equal to zero we can find the maximum speed that the car can have and still stay on the track.

$$ F_g = m\frac{v_{\text{max}}^2}{R} \quad (4) $$
\[ mg = \frac{v_{\text{max}}^2}{R} \quad (5) \]
\[ \Rightarrow v_{\text{max}} = \sqrt{gR} \quad (6) \]

Now that we know the maximum velocity at the top of the hill, we can use conservation of energy to find the starting height.

\[ K_f + U_f = K_i + U_i \quad (7) \]
\[ \frac{1}{2}mv_{\text{max}}^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \quad (8) \]
\[ \frac{1}{2}mv_{\text{max}}^2 + mgy_f = 0 + mgy_i \quad (9) \]

We’ll use \( y_f = R \) for the height at the top of the hill and factor the mass term,

\[ \frac{1}{2}v_{\text{max}}^2 + gR = 0 + gy_i \quad (10) \]
\[ y_i = \frac{v_{\text{max}}^2}{2g} + R \quad (11) \]

We can now plug in our results for \( v_{\text{max}} \) from above

\[ y_i = \frac{(\sqrt{gR})^2}{2g} + R \quad (12) \]
\[ = \frac{R}{2} + R \quad (13) \]
\[ = \frac{3R}{2} \quad (14) \]

(b) Plugging into the above equation, \( h_{\text{max}} \) for a roller coaster that has \( R = 10 \text{ m} \) we get

\[ y_i = \frac{3R}{2} = \frac{3 \times 10 \text{ m}}{2} = 15 \text{ m} \quad (15) \]
9. A sled starts from rest at the top of the frictionless, hemispherical, snow-covered hill shown below.

(a) Find an expression for the sled’s speed when it is at angle $\phi$.

(b) Use Newton’s laws to find the maximum speed the sled can have at angle $\phi$ without leaving the surface.

(c) At what angle $\phi_{\text{max}}$ does the sled “fly off” the hill?

Solution:

(a) Find sled’s speed When the sled is at a position that relates to angle $\phi$, the height is the $R\cos\phi$. Using conservation of energy to find the speed,

$$K_f + U_f = K_i + U_i$$  

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$  

$$\frac{1}{2}(v_f)^2 + gR\cos\phi = 0 + gy_i$$

$$(v_f)^2 = 2gR - 2gR\cos\phi = 2gR(1 - \cos\phi)$$

$$v_f = \sqrt{2gR(1 - \cos\phi)}$$

(b) Find max speed

$$\sum F = ma = -m\frac{v^2}{R}$$

$$F_N - F_g\cos\phi = -m\frac{v^2}{R}$$
The critical speed is just as the normal force equals zero. Factoring the negatives and the masses, and setting \( F_g = mg \) and \( F_N = 0 \),

\[
g \cos \phi = \frac{(v_c)^2}{R} \quad \text{(8)}
\]

\[
v_c = \sqrt{gR \cos \phi} \quad \text{(9)}
\]

(c) Find the angle when the sled gets air! We can set the speeds from the two previous parts equal to each other to find \( \phi \).

\[
\sqrt{gR \cos \phi} = \sqrt{2gR(1 - \cos \phi)} \quad \text{(10)}
\]

\[
gR \cos \phi = 2gR(1 - \cos \phi) \quad \text{(11)}
\]

\[
\cos \phi = 2 - 2 \cos \phi \quad \text{(12)}
\]

\[
3 \cos \phi = 2 \quad \text{(13)}
\]

\[
\phi = \cos^{-1} \left( \frac{2}{3} \right) \to \phi = 48^\circ \quad \text{(14)}
\]
10. Bob can throw a 500 g rock with a speed of 30 m/s. He moves his hand forward 1.0 m while doing so.

(a) How much work does Bob do on the rock?
(b) How much force, assumed to be constant, does Bob apply to the rock?
(c) What is Bob’s maximum power output as he throws the rock?

Solution:

(a) Calculate work

\[
W = \Delta K
\]

\[
W = \frac{1}{2} m(v_f)^2 - \frac{1}{2} m(v_i)^2
\]

\[
W = \frac{1}{2} m(v_f)^2 - 0
\]

\[
W = \frac{1}{2} 0.5 \text{kg}(30 \text{m/s})^2
\]

\[
W = 225 \text{ J}
\]

(b) Calculate force

\[
W = F \cdot \Delta r = F \Delta r
\]

\[
F = \frac{W}{\Delta r} = \frac{225 \text{ J}}{1 \text{m}}
\]

\[
F = 225 \text{ N}
\]

(c) Calculate power

\[
P = F \cdot v = Fv
\]

\[
P = 225 \text{ N}(30 \text{m/s})
\]

\[
P = 6750 \text{ W}
\]
11. Truck brakes can fail if they get too hot. In some mountainous areas, ramps of loose gravel are constructed to stop runaway trucks that have lost their brakes. The combination of a slight upward slope and a large coefficient of rolling resistance as the truck tires sink into the gravel brings the truck safely to a halt. Suppose a gravel ramp slopes upward at $6.0^\circ$ and the coefficient of rolling friction is 0.40. Use work and energy to find the length of a ramp that will stop a 15,000 kg truck that enters the ramp at 35 m/s ($\approx 75$ mph).

**Solution:**

We’ll identify the truck and the loose gravel as the system. We need the gravel inside the system because friction increases the temperature of the truck and the gravel. We will also use the model of kinetic friction and the conservation of energy equation.

\[
K_f + U_f + \Delta E_{th} = K_i + U_i + W_{ext} \quad (1)
\]
\[
0 + U_f + \Delta E_{th} = K_i + 0 + 0 \quad (2)
\]

The thermal energy created by friction is

\[
\Delta E_{th} = f_k(\Delta x) \quad (3)
\]
\[
= (\mu_k F_N)(\Delta x) \quad (4)
\]
\[
= (\mu_k mg \cos \theta)(\Delta x) \quad (5)
\]
\[
= (\mu_k mg \cos \theta)(\Delta x) \quad (6)
\]

Using geometry, the final gravitational potential energy of the truck is

\[
U_f = mgy_f \quad (7)
\]
\[
= mg(\Delta x) \sin \theta \quad (8)
\]

Finally, the initial kinetic energy is simply

\[
K_i = \frac{1}{2}mv_i^2 \quad (9)
\]
Plugging everything into the conservation of energy equation, we get

\[ mg(\Delta x) \sin \theta + \mu_k mg \cos \theta (\Delta x) = \frac{1}{2} mv_i^2 \]  

(10)

\[ g\Delta x (\sin \theta + \mu_k \cos \theta) = \frac{1}{2} v_i^2 \]  

(11)

\[ \Rightarrow \Delta x = \frac{v_i^2}{2g(\sin \theta + \mu_k \cos \theta)} \]  

(12)

\[ = \frac{(35 \text{ m/s})^2}{2 \times (9.8 \text{ m/s}^2) \times (\sin 6^\circ + 0.4 \times \cos 6^\circ)} \]  

(13)

\[ = 124 \text{ m} \]  

(14)

\[ = 0.12 \text{ km.} \]  

(15)
12. An 8.0 kg crate is pulled 5.0 m up a 30° incline by a rope angled 18° above the incline. The tension in the rope is 120 N, and the crate’s coefficient of kinetic friction on the incline is 0.25.

(a) How much work is done by tension, by gravity, and by the normal force?
(b) What is the increase in thermal energy of the crate and incline?

Solution:

(a) Find work done by tension, gravity and the normal

\[ W_T = T \cdot \Delta r = T \Delta x \cos(18^\circ) \]
\[ = (120 \text{ N}) \times (5.0 \text{ m}) \times (\cos 18^\circ) \]
\[ = 570 \text{ J} \]

\[ W_g = F_g \cdot \Delta r = mg \Delta x \cos 120^\circ \]
\[ = (6 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (5.0 \text{ m}) \times (\cos 120^\circ) \]
\[ = -196 \text{ J} \]

\[ W_N = 0 \text{ J} \]

(b) Find the increase in thermal energy

\[ \Delta E_{th} = f_k \cdot \Delta r = F_x \Delta x = \mu_k F_n \Delta x \]

To find the normal force,

\[ \sum F_y = 0 \]
\[ F_n - F_y \cos 30^\circ + T \sin 18^\circ = 0 \]
\[ \Rightarrow F_n = F_y \cos(30^\circ) - T \sin 18^\circ \]
\[ = (8.0 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (\cos 30^\circ) - (120 \text{ N}) \times (\sin 18^\circ) \]
\[ = 30.814 \text{ N} \]
Plugging this result into the equation for the thermal energy, we find

\[
\Delta E_{th} = \mu_k F_n \Delta x \tag{15}
\]
\[
= (0.25) \times (30.814 \text{ N}) \times (5.0 \text{ m}) \tag{16}
\]
\[
= 38.5 \text{ J} \tag{17}
\]
13. A 5.0 kg cat leaps from the floor to the top of a 95 cm high table. If the cat pushes against the floor for 0.20 s to accomplish this feat, what was her average power output during the pushoff period?

**Solution:**

The average power output during the push-off period is equal to the work done by the cat divided by the time the cat applied the force. Since the force on the floor by the cat is equal in magnitude to the force on the cat by the floor, work done by the cat can be found using the work-kinetic-energy theorem during the push-off period: $W_{\text{net}} = W_{\text{floor}} = \Delta K$. We do not need to explicitly calculate $W_{\text{cat}}$, since we know that the cat’s kinetic energy is transformed into its potential energy during the leap. In other words,

$$\Delta U_g = mg(y_2 - y_1)$$

$$= (5.0 \text{ kg})(9.8 \text{ m/s}^2)(0.95 \text{ m})$$

$$= 46.55 \text{ J.}$$

Thus, the average power output during the push-off period is

$$P = \frac{W_{\text{net}}}{t}$$

$$= \frac{46.55 \text{ J}}{0.20 \text{ s}}$$

$$= 0.23 \text{ kW.}$$
14. In a hydroelectric dam, water falls 25 m and then spins a turbine to generate electricity.

(a) What is $\Delta U$ of 1.0 kg of water?

(b) Suppose the dam is 80% efficient at converting the water’s potential energy to electrical energy. How many kilograms of water must pass through the turbines each second to generate 50 MW of electricity?

Solution:

(a) The change in the potential energy of 1.0 kg of water falling 25 m is

\[
\Delta U_g = -mgh \\
= -(1.0 \text{ kg})(9.8 \text{ m/s}^2)(25 \text{ m}) \\
= -250 \text{ J}. 
\]

(b) The dam is required to produce 50 MW = $50 \times 10^6$ W of power every second, or in other words $50 \times 10^6$ J of energy per second. If the dam is 80% efficient at converting the water’s potential energy into electrical energy, then every kilogram of water that falls produces $0.8 \times 250 \text{ J} = 200 \text{ J}$ of electrical energy. Therefore, the total mass of water needed every second is

\[
50 \times 10^6 \text{ J} \times \frac{1.0 \text{ kg}}{200 \text{ J}} = 2.5 \times 10^5 \text{ kg}. 
\]

This is a typical value for a small hydroelectric dam.
15. You need to determine the density of a ceramic statue. If you suspend it from a spring scale, the scale reads 28.4 N. If you then lower the statue into a tub of water, so that it is completely submerged, the scale reads 17.0 N. What is the statue’s density?

**Solution:**

The buoyant force, $F_B$, on the can is given by Archimedes’ principle.

The free-body diagram sketched above shows that the gravitational force of the statue, $F_{G,\text{statue}}$ is balanced by the spring force, $F_{sp}$, and the buoyant force, $F_B$ exerted by the water. Since the statue is in static equilibrium, we know that the net force is zero. The rest of the solution follows through a series of substitutions, recalling that the density of water is $\rho_w = 1000 \text{ kg m}^{-3}$:

\[
F_B + F_{sp} - F_{G,\text{statue}} = 0 \quad \Rightarrow \quad F_B = F_{G,\text{statue}} - F_{sp} \quad (1)
\]

\[
\rho_w V_{\text{statue}}g = F_{G,\text{statue}} - F_{sp} \quad (2)
\]

\[
V_{\text{statue}} = \frac{F_{G,\text{statue}} - F_{sp}}{\rho_w g} \quad (3)
\]

\[
m_{\text{statue}} = \frac{F_{G,\text{statue}} - F_{sp}}{\rho_w g} \quad (4)
\]

\[
\frac{\rho_{\text{statue}}}{m_{\text{statue}}} = \frac{\rho_w g}{F_{G,\text{statue}} - F_{sp}} \quad (5)
\]

\[
\rho_{\text{statue}} = \frac{\rho_w g}{m_{\text{statue}} F_{G,\text{statue}} - F_{sp}} \quad (6)
\]

\[
\rho_{\text{statue}} = \frac{\rho_w F_{G,\text{statue}}}{(1000 \text{ kg m}^{-3}) \times (28.4 \text{ N})} \quad (7)
\]

\[
= \frac{(28.4 \text{ N} - 17.0 \text{ N})}{(1000 \text{ kg m}^{-3})} \quad (8)
\]

\[
= 2490 \text{ kg m}^{-3}. \quad (9)
\]
16. A 355 mL soda can is 6.2 cm in diameter and has a mass of 20 g. Such a soda can half full of water is floating upright in water. What length of the can is above the water level?

Solution:

The buoyant force, $F_B$, on the can is given by Archimedes’ principle.

Let the length of the can above the water level be $d$, the total length of the can be $L$, and the cross-sectional area of the can be $A$. The can is in static equilibrium, so from the free-body diagram sketched above we have

\[ \sum F_y = F_B - F_{G,\text{can}} - F_{G,\text{water}} = 0 \]

\[ \rho_{\text{water}} A (L - d) g = (m_{\text{can}} + m_{\text{water}}) g \]  \hspace{1cm} (2)

\[ L - d = \frac{(m_{\text{can}} + m_{\text{water}})}{A \rho_{\text{water}}} \]  \hspace{1cm} (3)

\[ \Rightarrow d = L - \frac{(m_{\text{can}} + m_{\text{water}})}{A \rho_{\text{water}}} \]  \hspace{1cm} (4)

Recalling that one liter equals $10^{-3}$ m$^3$ and the density of water is $\rho_{\text{water}} = 1000$ kg m$^{-3}$, the mass of the water in the can is

\[ m_{\text{water}} = \rho_{\text{water}} \left( \frac{V_{\text{can}}}{2} \right) \]  \hspace{1cm} (5)

\[ = (1000 \text{ kg m}^{-3}) \times \left( \frac{355 \times 10^{-6} \text{ m}^3}{2} \right) \]  \hspace{1cm} (6)

\[ = 0.1775 \text{ kg.} \]  \hspace{1cm} (7)

To find the length of the can, $L$, we have:

\[ V_{\text{can}} = AL \]  \hspace{1cm} (8)

\[ \Rightarrow L = \frac{355 \times 10^{-6} \text{ m}^3}{\pi (0.031 \text{ m})^2} \]  \hspace{1cm} (9)

\[ = 0.1176 \text{ m.} \]  \hspace{1cm} (10)
Finally, substituting everything into equation (4), we get:

\[
\begin{align*}
  d &= 0.1176 \text{ m} - \frac{(0.020 \text{ kg} + 0.1775 \text{ kg})}{\pi \times (0.031 \text{ m})^2 \times (1000 \text{ kg m}^{-3})} \quad (11) \\
  &= 0.0522 \text{ m} \quad (12) \\
  &= 5.2 \text{ cm.} \quad (13)
\end{align*}
\]
17. Water flowing out of a 16 mm diameter faucet fills a 2.0 L bottle in 10 s. At what distance below the faucet has the water stream narrowed to 10 mm in diameter?

Solution:

We treat the water as an ideal fluid obeying Bernoulli’s equation. The pressure at point 1 is $p_1$ and the pressure at point 1 is $p_2$. Both $p_1$ and $p_2$ are atmospheric pressure. The velocity and the area at point 1 are $v_1$ and $A_1$, and at point 2 they are $v_2$ and $A_2$. Let $d$ be the distance of point 2 below point 1.

First, we use the time it takes to fill a 2.0 L bottle to compute the flow rate, $Q$, the rate at which water is flowing out of the faucet, remembering that one liter equals $10^{-3}$ m$^3$:

$$Q = \frac{2.0 \text{ L} \times (10^{-3} \text{ m}^3/\text{L})}{10 \text{ s}}$$

$$= 2.0 \times 10^{-4} \text{ m}^3/\text{s}.$$  

Next, we use this result to find the velocity, $v_1$, with which the water leaves the faucet through the $D_1 = 16 \times 10^{-3}$ m diameter aperture (at point 1):

$$Q = v_1 A_1$$

$$\Rightarrow v_1 = \frac{Q}{A_1}$$

$$= \frac{Q}{\pi (D_1/2)^2}$$

$$= \frac{2.0 \times 10^{-4} \text{ m}^3/\text{s}}{\pi \times (8 \times 10^{-3} \text{ m})^2}$$

$$= 1.0 \text{ m/s}.$$
Now, from the continuity equation we have

\[ v_2 A_2 = v_1 A_1 \]  

⇒ \[ v_2 = v_1 \frac{A_1}{A_2} \]  

\[ = v_1 \frac{\pi(D_1/2)^2}{\pi(D_2/2)^2} \]  

\[ = v_1 \left( \frac{D_1}{D_2} \right)^2 \]  

\[ = 1.0 \text{ m/s} \left( \frac{16 \times 10^{-3} \text{ m}}{10 \times 10^{-3} \text{ m}} \right)^2 \]  

\[ = 2.56 \text{ m/s.} \]  

Finally, we turn to Bernoulli’s equation to find the height, \( d \):

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]  

\[ \rho g(y_1 - y_2) = \frac{1}{2} \rho (v_2^2 - v_1^2) \]  

\[ gd = \frac{1}{2} (v_2^2 - v_1^2) \]  

\[ d = \frac{(v_2^2 - v_1^2)}{2g} \]  

\[ = \frac{(2.56 \text{ m/s})^2 - (1.0 \text{ m/s})^2}{2 \times 9.8 \text{ m/s}^2} \]  

\[ = 0.283 \text{ m} \]  

\[ \approx 28 \text{ cm.} \]
18. A hurricane wind blows across a 6.0 m \times 15.0 m flat roof at a speed of 130 km/hr.

(a) Is the air pressure above the roof higher or lower than the pressure inside the house? Explain.

(b) What is the pressure difference?

(c) How much force is exerted on the roof? If the roof cannot withstand this much force, will it “blow in” or “blow out”?

**Solution:**

(a) The pressure above the roof is lower due to the higher velocity of the air.

(b) Bernoulli’s equation with $y_{\text{inside}} \approx y_{\text{outside}}$ is

\[
p_{\text{inside}} = p_{\text{outside}} + \frac{1}{2} \rho_{\text{air}} v^2
\]

\[
\Rightarrow \Delta p = \frac{1}{2} \rho_{\text{air}} v^2
\]

\[
= \frac{1}{2} \times (1.28 \text{ kg/m}^3) \times \left( \frac{130 \text{ km/hr}}{1 \text{ hr}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \right)^2
\]

\[
= 835 \text{ Pa.}
\]

The pressure difference is 0.83 kPa.

(c) The force on the roof is

\[
F_{\text{roof}} = (\Delta p) A
\]

\[
= (835 \text{ Pa}) \times (6.0 \text{ m} \times 15.0 \text{ m})
\]

\[
= 7.5 \times 10^4 \text{ N.}
\]

The roof will blow up, because the pressure inside the house is greater than the pressure on the top of the roof.
19. A gas is compressed from 600 cm\(^3\) to 200 cm\(^3\) at a constant pressure of 400 kPa. At the same time, 100 J of heat energy is transferred out of the gas. What is the change in thermal energy of the gas during this process?

**Solution:**
This is a first law of thermodynamics problem involving an *isobaric* (constant pressure) process. We know that \(W > 0\) because the gas is compressed, which transfers energy into the system. Also, 100 J of heat is transferred out of the gas. The first law of thermodynamics gives

\[
\Delta E_{th} = W + Q
\]

\[
= -p\Delta V + Q
\]

\[
= -(4.0 \times 10^5 \text{ Pa}) \times (200 - 600) \times 10^{-6} \text{ m}^3 - 100 \text{ J}
\]

\[
= 60 \text{ J.}
\]

Therefore, the thermal energy increases by 60 J.
20. A 100 g ice cube at \(-10^\circ\) C is placed in an aluminum cup whose initial temperature is \(70^\circ\) C. The system comes to an equilibrium temperature of \(20^\circ\) C. What is the mass of the cup?

**Solution:**

There are two interacting systems: aluminum and ice. The system comes to thermal equilibrium in four steps: (1) the ice temperature increases from \(-10^\circ\) C to \(0^\circ\) C; (2) the ice becomes water at \(0^\circ\) C; (3) the water temperature increases from \(0^\circ\) C to \(20^\circ\) C; and (4) the cup temperature decreases from \(70^\circ\) C to \(20^\circ\) C.

Since the aluminum and ice form a closed system, we have

\[
Q = Q_1 + Q_2 + Q_3 + Q_4 = 0. \tag{1}
\]

Each term is as follows:

\[
Q_1 = M_{\text{ice}} c_{\text{ice}} \Delta T = (0.100 \text{ kg}) \times [2090 \text{ J/(kg K)}] \times (10 \text{ K}) = 2090 \text{ J}.
\]

\[
Q_2 = M_{\text{ice}} L_f = (0.100 \text{ kg}) \times (3.33 \times 10^5 \text{ J/kg}) = 33,300 \text{ J}.
\]

\[
Q_3 = M_{\text{ice}} c_{\text{water}} \Delta T = (0.100 \text{ kg}) \times [4190 \text{ J/(kg K)}] \times (20 \text{ K}) = 8380 \text{ J}.
\]

\[
Q_4 = M_{\text{Al}} c_{\text{Al}} \Delta T = (45,000 \text{ J/kg}) \times (-50 \text{ K}) = -22500 \text{ J}.
\]

Inserting everything into equation (1), we get

\[
43,770 \text{ J} - (45,000 \text{ J/kg})M_{\text{Al}} = 0 \Rightarrow M_{\text{Al}} = 0.97 \text{ kg}. \tag{6}
\]
Kinematics and Mechanics

\[ x_f = x_i + v_{xi}t + \frac{1}{2}a_xt^2 \]
\[ v_{xf} = v_{xi} + a_xt \]
\[ v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \]
\[ y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2 \]
\[ v_{yf} = v_{yi} + a_yt \]
\[ v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i) \]
\[ \theta_f = \theta_i + \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2 \]
\[ \omega_f = \omega_i + \alpha\Delta t \]
\[ \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \]
\[ s = rt \]
\[ c = 2\pi r \]
\[ v = \frac{2\pi r}{T} \]

Forces

\[ \vec{F}_{\text{net}} = \Sigma \vec{F} = ma \]
\[ \vec{F}_{\text{net}} = \Sigma \vec{F}_r = ma = m\frac{v^2}{r} \]
\[ F_g = mg \]
\[ 0 < f_k \leq \mu_s F_N \]
\[ f_k = \mu_k F_N \]
\[ \vec{F}_{AonB} = -\vec{F}_{BonA} \]

Momentum

\[ \vec{p}_i = \vec{p}_f \]
\[ \vec{p} = m\vec{v} \]
\[ (v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2} (v_{ix})_1 \]
\[ (v_{fx})_2 = \frac{2m_1}{m_1 + m_2} (v_{ix})_1 \]

Energy

\[ K_f + U_{gf} = K_i + U_{gi} \]
\[ \Delta K + \Delta U + \Delta E_{th} = \Delta E_{mech} + \Delta E_{th} = \Delta E_{sys} = W_{ext} \]
\[ K_f + U_f + \Delta E_{th} = K_i + U_i + W_{ext} \]
\[ K = \frac{1}{2}mv^2 \]
\[ U = mgy \]
\[ \Delta E_{th} = f_k \Delta s \]
\[ W = \vec{F} \cdot \vec{d} \]
\[ P = \vec{F} \cdot \vec{v} \]
\[ P = \Delta E/\Delta t \]

Fluids and Thermal Energy

\[ P = \frac{F}{A} \]
\[ \rho = \frac{m}{V} \]
\[ Q = \rho A \]
\[ v_1 A_1 = v_2 A_2 \]
\[ p_1 + \frac{1}{2} \rho v^2_1 + \rho gy_1 = p_2 + \frac{1}{2} \rho v^2_2 + \rho gy_2 \]
\[ Q_{\text{net}} = Q_1 + Q_2 + ... = 0 \]
\[ Q = ML_f \] for freezing/melting
\[ Q = \rho c \Delta T \]

Constants

\[ g = 9.8 \text{ m/s}^2 \]
\[ M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg} \]
\[ r_{\text{earth--sun}} = 1.50 \times 10^{11} \text{ m} \]
\[ \rho_{\text{water}} = 1000 \text{ kg/m}^3 \]
\[ \rho_{\text{air}} = 1.28 \text{ kg/m}^3 \]
\[ c_{\text{ice}} = 2090 \text{ J/kg K} \]
\[ c_{\text{water}} = 4190 \text{ J/kg K} \]
\[ c_{\text{Al}} = 900 \text{ J/kg K} \]
\[ L_f = 333,000 \text{ J/kg ice to water} \]