Chapter 11
Vibrations and Waves

11-1 Simple Harmonic Motion

We assume that the surface is frictionless. There is a point where the spring is neither stretched nor compressed; this is the equilibrium position. We measure displacement from that point ($x = 0$ on the previous figure).

The force exerted by the spring depends on the displacement:

$$ F = -kx \quad (11-1) $$

If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic. The mass and spring system is a useful model for a periodic system.
11-1 Simple Harmonic Motion

- Displacement is measured from the equilibrium point
- Amplitude is the maximum displacement
- A cycle is a full to-and-fro motion; this figure shows half a cycle
- Period is the time required to complete one cycle
- Frequency is the number of cycles completed per second

If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator.

11-2 Energy in the Simple Harmonic Oscillator

We already know that the potential energy of a spring is given by:

$$PE = \frac{1}{2}kx^2$$

The total mechanical energy is then:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad (11-3)$$

The total mechanical energy will be conserved, as we are assuming the system is frictionless.
11-2 Energy in the Simple Harmonic Oscillator

If the mass is at the limits of its motion, the energy is all potential.

If the mass is at the equilibrium point, the energy is all kinetic.

We know what the potential energy is at the turning points:

$$E = \frac{1}{2} k A^2 \quad (11-4a)$$

The total energy is, therefore

$$\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \quad (11-4c)$$

And we can write:

This can be solved for the velocity as a function of position:

$$v = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}} \quad (11-5)$$

where

$$v_{\text{max}} = \left(\frac{k}{m}\right) A$$

11-3 The Period and Sinusoidal Nature of SHM

If we look at the projection onto the x axis of an object moving in a circle of radius A at a constant speed $v_{\text{max}}$, we find that the x component of its velocity varies as:

$$v = v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}}$$

This is identical to SHM.

Therefore, we can use the period and frequency of a particle moving in a circle to find the period and frequency:

$$T = \frac{2\pi r}{v}$$

$$r = A$$

$$v = \sqrt{\frac{k}{m}} A$$

Simplifying gives:

$$T = \frac{2\pi A}{\sqrt{\frac{k}{m} A}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}} A}$$
11-3 The Period and Sinusoidal Nature of SHM

We can similarly find the position as a function of time:

\[ x = A \cos \omega t \]  \hspace{1cm} (11-8a)

\[ = A \cos(2\pi ft) \]  \hspace{1cm} (11-8b)

\[ = A \cos(2\pi t/T) \]  \hspace{1cm} (11-8c)

The velocity and acceleration can be calculated as functions of time; the results are below, and are plotted at left.

\[ v = -v_{\text{max}} \sin \omega t \]  \hspace{1cm} (11-9)

\[ v_{\text{max}} = A \sqrt{\frac{k}{m}} \]

\[ a = -a_{\text{max}} \cos(2\pi t/T) \]  \hspace{1cm} (11-10)

\[ a_{\text{max}} = \frac{kA}{m} \]

Think-Pair-Share

- Problem 9: A 0.60-kg mass at the end of a spring vibrates 3.0 times per second with an amplitude of 0.13 m. Determine (a) the velocity when it passes the equilibrium point, (b) the velocity when it is 0.10 m from equilibrium, (c) the total energy of the system, and (d) the equation describing the motion of the mass, assuming that \( x \) was a maximum at \( t=0 \).
11-4 The Simple Pendulum

A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.

In order to be in SHM, the restoring force must be proportional to the negative of the displacement. Here we have:

\[ F = -mg \sin \theta \]

which is proportional to \( \sin \theta \) and not to \( \theta \) itself.

However, if the angle is small, \( \sin \theta \approx \theta \).

Therefore, for small angles, we have:

\[ F \approx -\frac{mg}{L} \theta \]

where \( \theta = L \theta \)

The period and frequency are:

\[ T = 2\pi \sqrt{\frac{L}{g}} \quad \text{(11-11a)} \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad \text{(11-11b)} \]

So, as long as the cord can be considered massless and the amplitude is small, the period does not depend on the mass.
Think-Pair-Share

• Problem 29: How long must a simple pendulum be if it is to make exactly one swing per second? (That is, one complete vibration takes exactly 2.0 s.)